

SOLUTIONS & ANSWERS FOR AIEEE-2011 VERSION – R

Part – A – Mathematics

1. Ans: $\left[0, \frac{1}{2}\right]$

Sol: $1 - P^5 \geq \frac{31}{32}$

$P^5 \leq 1 - \frac{31}{32}$

$\leq \frac{1}{32}$

$P \leq \frac{1}{2} = \left[0, \frac{1}{2}\right]$

2. Ans: -144

Sol: $(1 - x - x^2 + x^3)^6 = (1 - x)^6 (1 - x^2)^6$
 $= (1 - 6x + \dots - 20x^3 \dots - 6x^5) x$
 $(1 - 6x^2 + 75x^4 - 20x^6 \dots)$
 $= 120 - 300 + 36$
 $= 156 - 300 = -144$

3. Ans: Does not exist

Sol: $\lim_{x \rightarrow 2} \sqrt{2} \left| \frac{\sin(x-2)}{(x-2)} \right|$

Limit does not exist

4. Ans: Statement-1 is true, Statement-2 is true;
Statement-2 is **not** a correct explanation
for Statement-1.

Sol: A = (x, y) y - x ∈ Z
 B = (x, y) x = αy for rational α
 A : x - x = 0 ∈ Z ⇒ (x, x) ∈ A reflexive
 y - x ∈ Z ⇒ x - y ∈ Z
 ⇒ (y, x) ∈ A symmetric
 y - x ∈ Z and z - y ∈ Z ⇒ z - x ∈ Z
 ∴ (x, z) ∈ A transitive
 A is equivalence relation
 Statement - 1 is true
 B: x = 1, x ⇒ (x, x) ∈ B reflexive
 x = αy ⇒ y = $\frac{1}{\alpha} x$ ∴ (y, x) ∈ B
 symmetric
 x = αy and y = αz ⇒ x = α² z
 ∴ (x, z) ∈ B transitive
 B is equivalence relation
 Statement - 2 is true but I does not
 follow from 2.

5. Ans: β ∈ (1, ∞)

Sol: If 1 + ai is root (a, real)
 Then (1 + i a)² + α (1 + i a) + β = 0
 2a + α = 0 ⇒ α = -2 a ≠ 0
 1 - a² + α + β = 0

$1 - a^2 + \beta = 0$
 $\beta = a^2 + 1 > 1 \therefore \beta \in (1, \infty)$

6. Ans: $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

Sol: $\frac{d^2x}{dy^2} = \frac{d}{dy} \left(\frac{dx}{dy} \right)$
 $= \frac{d}{dy} \left[\frac{1}{\frac{dy}{dx}} \right]$
 $= \frac{-1}{\left(\frac{dy}{dx}\right)^2} \cdot \frac{d}{dy} \left(\frac{dy}{dx} \right)$
 $= \frac{-1}{\left(\frac{dy}{dx}\right)^2} \frac{d^2y}{dx^2} \left(\frac{dx}{dy} \right)$
 $= -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

7. Ans: 2

Sol: $\begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$
 $4(4-2) - k(k-2) + 2(2k-8) = 0$
 $= 8 - k^2 + 2k + 4k - 16 = 0$
 $\Rightarrow -k^2 + 6k - 8 = 0$
 $k^2 - 6k + 8 = 0$
 $\Rightarrow (k-4)(k-2) = 0$
 $\Rightarrow k = 2, 4$
 $\therefore k = 2$

8. Ans: Statement-1 is true, Statement-2 is true;
Statement-2 is **not** a correct explanation
for Statement-1.

Sol: A (1, 0, 7) B (1, 6, 3)
 $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{5}$
 P (λ, 2λ + 1, 3λ + 2)
 drs (λ - 1, 2λ + 1, 3λ - 5)
 $\therefore \lambda - 1 + 2(2\lambda + 1) + 3(3\lambda - 5) = 0$
 $14\lambda - 14 = 0 \Rightarrow \lambda = 1$
 P (1, 3, 5) is mid point of A and B
 Statement-1 is true
 Statement-2 is also true but
 statement-1 does not follow from 2

9. Ans: $\sim (Q \leftrightarrow (P \wedge \sim R))$

Sol: The given statement is
 $(P \wedge \sim R) \leftrightarrow Q \equiv Q \leftrightarrow (P \wedge \sim R)$
 \therefore The required negative is
 $\sim [Q \leftrightarrow (P \wedge \sim R)]$

10. Ans: Statement-1 is true, Statement-2 is false.

Sol: P is (-2, -2) and Q (-1, 2) since R bisect
 $\angle POQ$, $PR \perp RQ = OP : OQ$
 $= \sqrt{4+4} : \sqrt{1+4} = \sqrt{8} : \sqrt{5}$
 \therefore Statement 1 is true
 But statement 2 is false.

11. Ans: 21 months

Sol: Total savings = 11040
 Savings in the first 2 months = 400
 Hence, savings in the next n months
 = 10640

We have

$$\frac{n}{2} [400 + (n-1)40] = 10640$$

$$[200 + (n-1)20]n = 10640$$

$$200n + 20n^2 - 20n = 10640$$

$$20n^2 + 180n - 10640 = 0$$

$$\frac{n^2 + 9n - 532 = 0}{n = \frac{9 \pm \sqrt{81 + 2128}}{2}}$$

$$= \frac{-9 \pm \sqrt{2209}}{2} = \frac{-9 \pm 47}{2}$$

$$= 19$$

Therefore, answer is 21 months

12. Ans: $3x^2 + 5y^2 - 32 = 0$

Sol: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{9}{a^2} + \frac{1}{b^2} = 1$$

$$\frac{1}{b^2} = 1 - \frac{9}{a^2}$$

$$\frac{1}{a^2 \left(1 - \frac{9}{a^2}\right)} = \frac{a^2 - 9}{a^2}$$

$$a^2 - 9 = \frac{3}{5}$$

$$a^2 = 9 + \frac{3}{5} = \frac{32}{5}$$

$$b^2 = a^2 \times \frac{3}{5} = \frac{32}{5} \times \frac{3}{5} = \frac{32}{5}$$

Equation of the ellipse is

$$\frac{x}{\frac{32}{5}} + \frac{y^2}{\frac{32}{5}} = 1$$

$$3x^2 + 5y^2 - 32 = 0$$

13. Ans: $\frac{3}{4} \leq A \leq 1$

Sol: $A = \sin^2 x + \cos^4 x$
 $= \cos^4 x - \cos^2 x + 1$
 $= \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4}$

$$\therefore \frac{3}{4} \leq A \leq 1$$

14. Ans: $\pi \log 2$

Sol: $I = 8 \int_0^1 \frac{\log(1+x)}{1+x^2} dx$
 $= 8 \int_0^{\pi/4} \text{Log}(1 + \tan \theta) d\theta$
 $= \pi \log 2$

15. Ans: $\frac{2}{3}$

Sol: The angle is $\sin^{-1} \frac{3}{\sqrt{14}}$
 $\therefore \frac{1+4+3\lambda}{\sqrt{(1+4+\lambda^2)(1+4+9)}} = \frac{3}{\sqrt{14}}$
 $14(3\lambda+5)^2 = 9 \times 14(5+\lambda^2)$
 $9\lambda^2 + 30\lambda + 25 = 9\lambda^2 + 45$
 $\Rightarrow 30\lambda = 20 \Rightarrow \lambda = \frac{2}{3}$

16. Ans: local maximum at π and local minimum at 2π

Sol: $f(x) = \sqrt{x} \sin x$
 $f'(x) = \frac{2x \cos x + \sin x}{2\sqrt{x}}$
 $f'(x) = 0 \Rightarrow x = n\pi, n \in \mathbb{Z}$
 ie., $x = \pi, 2\pi$ in $(0, 5\pi/2)$
 $f''(\pi) < 0$ and $f''(2\pi) > 0$
 $\therefore f(x)$ has maximum at $x = \pi$
 And minimum at $x = 2\pi$

17. Ans: $(-\infty, 0)$

Sol: $|x| - x > 0$
 $\Rightarrow |x| > x$
 $\Rightarrow x \in (-\infty, 0)$

18. Ans: 4

Sol: Median = $\frac{25a+26b}{2}$
 $= \frac{51a}{2}$

Numerical value of the sum of the derivation

$$= \left| 2a \left\{ \frac{1}{2} + \frac{3}{2} + \frac{5}{2} + \dots + \frac{49}{2} \right\} \right|$$

$$= \left| \frac{2a \times 25^2}{2} \right| = |25^2 a|$$

$$\text{Mean derivation about median} = \left| \frac{25^2 a}{50} \right|$$

$$\left| \frac{25^2 a}{50} \right| = 50$$

$$|a| = \frac{50 \times 50}{25 \times 25} = 4$$

19. Ans: -5

$$\begin{aligned} \text{Sol: } |a| = |b| = 1 \quad a, b = 0 \\ (2a - b) \cdot ((a \times b) \times (a + 2b)) \\ = (2a - b) \times \\ [(a \cdot a) b - (a \cdot b) a + (2b \cdot a) b - (2b \cdot b)] \\ (2a - b) \cdot (b - 2a) = -5 \end{aligned}$$

20. Ans: $p = -\frac{3}{2}, q = \frac{1}{2}$

$$\text{Sol: } f(x) = \frac{\sin(p+1)x + \sin x}{x}, x < 0$$

$$= q, x = 0$$

$$\frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}}, x > 0$$

is continuous.

$$\Rightarrow p + 1 + 1 = q = \lim_{x \rightarrow 0} \frac{x}{x^{3/2}(\sqrt{x+x^2} + \sqrt{x})}$$

$$= \frac{1}{2}$$

$$\therefore p = -\frac{3}{2}, q = \frac{1}{2}$$

21. Ans: $|a| = c$

Sol: Two circle should touch each other

$$\text{Centres are } \left(\frac{a}{2}, 0\right) \text{ and } (0, 0)$$

\therefore also second circle passes through (0, 0)

$$\therefore c = a \Rightarrow |a| = c$$

22. Ans: $I - \frac{kT^2}{2}$

$$\text{Sol: } \frac{dv(t)}{dt} = -k(T-t)$$

$$V(t) = \int -k(T-t) dt$$

$$\frac{k(T-t)^2}{2} + C$$

$$t = 0, V(t) = I$$

$$\Rightarrow I = \frac{kT^2}{2} + C$$

$$C = I - \frac{kT^2}{2}$$

Therefore,

$$V(t) = \frac{k(T-t)^2}{2} + I - \frac{kT^2}{2}$$

$$\Rightarrow V(T) = 0 + I - \frac{kT^2}{2}$$

$$= I - \frac{kT^2}{2}$$

23. Ans: $P(C|D) \geq P(C)$

$$\begin{aligned} \text{Sol: } P(C|D) &= \frac{P(CD)}{P(D)} \\ &= \frac{P(C)}{P(D)} \\ &\geq P(C) \end{aligned}$$

24. Ans: Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

Sol: if $AB = BA$
 $(AB)^T = A^T B^T$
 $\Rightarrow AB$ is symmetric
 Statement-2 is true
 $(ABA)^T = A^T B^T A^T$
 Take $A = I$ and $B =$ some non-symmetric
 $\therefore ABA$ always
 $\therefore A(BA)$ and $(AB)A$ are symmetric
 Statement-1 is true but does not depend on Statement-2

25. Ans: (1, 1)

$$\begin{aligned} \text{Sol: } (1 + \omega)^7 &= A + B\omega \\ (-\omega^2)^7 &= A + B\omega \\ -\omega^{14} &= A + B\omega \\ -\omega^2 &= A + B\omega \\ 1 + \omega &= A + B\omega \\ \therefore A &= 1 \quad B = 1 \\ \therefore (1, 1) \end{aligned}$$

26. Ans: Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

$$\begin{aligned} \text{Sol: } x_1 + x_2 + x_3 + x_4 &= 6 \\ x_i &\geq 0 \\ \text{no. of ways} &= {}^9C_3 \\ S_2 &\text{ is true} \\ S_1 &\text{ is true} \\ S_1 &\text{ follows from } S_2 \end{aligned}$$

27. Ans: $\frac{3\sqrt{2}}{8}$

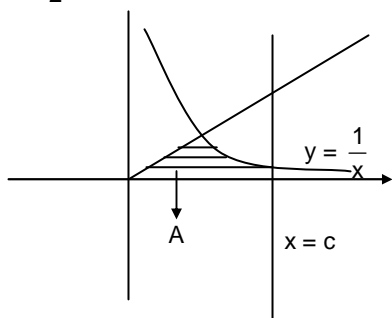
Sol: Slope of the line perpendicular to $y - x = 1$ is (-1)
 Hence $t = 1$

Point on the parabola corresponding to $t = 1$ is

$$\Rightarrow \left(\frac{1}{4}, \frac{1}{2}\right)$$

$$\therefore \text{shortest distance} = \frac{\frac{1}{4} - \frac{1}{2} + 1}{\sqrt{2}} = \frac{3\sqrt{2}}{8}$$

28. Ans: $\frac{3}{2}$ square units



Sol: $y = x$

$$y = \frac{1}{x} \Rightarrow x^2 = 1$$

$$\Rightarrow x = 1 \quad (x > 0)$$

$$y = \frac{1}{x}, x = e \Rightarrow x = e$$

$$\therefore \text{area } A = \int_1^e \left(x - \frac{1}{x}\right) dx$$

$$= \frac{e^2 - 1}{2} - \log e$$

$$= \frac{e^2 - 3}{2}$$

$$\text{Required area} = \frac{1}{2} \cdot e^2 - \frac{e^2 - 3}{2} = \frac{3}{2}$$

29. Ans: 7

Sol: $\frac{dy}{dx} = y + 3$

$$\frac{dy}{y+3} = dx$$

$$\log(y+3) = x + c$$

$$\therefore y + 3 = c e^x$$

$$x = 0, y = 2 \Rightarrow c = 5$$

$$\therefore y = 5 e^x - 3$$

$$\therefore y(\log 2) = 5 e^{\log 2} - 3 = 5 \times 2 - 3 = 7$$

30. Ans: $\bar{c} - \frac{\bar{a} \cdot \bar{c}}{\bar{a} \cdot \bar{b}} \bar{b}$

Sol: $\bar{b} \times \bar{c} = \bar{b} \times \bar{d}$

$$\bar{a} \cdot \bar{d} = 0$$

$$\bar{b} \times (\bar{c} - \bar{d}) = 0$$

\bar{b} and $(\bar{c} - \bar{d})$ are collinear

$$\bar{b} = k(\bar{c} - \bar{d})$$

$$\bar{a} \cdot \bar{b} = k(\bar{a} \cdot \bar{c} - \bar{a} \cdot \bar{d})$$

$$k \left[\bar{a} \cdot \bar{c} \right]$$

$$k = \frac{\bar{a} \cdot \bar{b}}{\bar{a} \cdot \bar{c}}$$

$$\bar{b} \cdot \bar{c} - \bar{d} = \frac{\bar{a} \cdot \bar{c}}{\bar{a} \cdot \bar{b}} \bar{b}$$

$$\bar{d} = \bar{c} - \frac{\bar{a} \cdot \bar{c}}{\bar{a} \cdot \bar{b}} \bar{b}$$

PART B – CHEMISTRY

31. Ans: Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series

Sol: All the lanthanoids does not exhibit +4 oxidation state.

32. Ans: A_2B_5

Sol: $A_1B_{5/2} = A_2B_5$

33. Ans: 2.82 BM

Sol: There are two unpaired electrons is $[NiCl_4]^{2-}$ hence the paramagnetic moment is 2.82 BM.

34. Ans: The complex is an outer orbital complex

Sol: $[Cr(NH_3)_6]Cl_3$ is not an outer orbital complex.

35. Ans: 32 times

Sol: 2 times increase for $10^\circ C$
 $2^5 = 32$ times increase for $50^\circ C$

36. Ans: a for $Cl_2 > a$ for C_2H_6 but b for $Cl_2 < b$ for C_2H_6

Sol: 'a' is a measure of attraction between the molecules and 'b' the size of the molecules.

37. Ans: sp^2, sp, sp^3

Sol: $NO_3^- - sp^2, NO_2^+ - sp$ and $NH_4^+ - sp^3$

38. Ans: 804.32 g

Sol: $\Delta T_f = K_f \times \frac{W_2}{M_2} \times \frac{1}{W_1}$

$$6 = 1.86 \times \frac{W_2}{62} \times \frac{1}{4}$$

$$W_2 = 800 \text{ g}$$

Wt. of glycol required is more than 800 g

39. Ans: $4f^7 5d^1 6s^2$

Sol: The outer electronic configuration of ${}_{64}\text{Gd}$ is $4f^7 5d^1 6s^2$

40. Ans: pentagonal bipyramid

Sol: IF_7 is pentagonal bipyramidal.

41. Ans: a vinyl group

Sol: Formation of HCHO in ozonolysis shows the presence of $\text{CH}_2 = \text{CH} -$ group.

42. Ans: $\alpha = \frac{i - 1}{(x + y - 1)}$

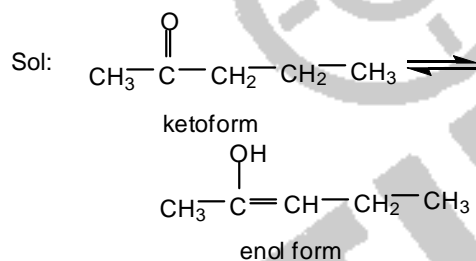
Sol: $i = 1 - \alpha + n\alpha$; $n = x + y$

$$\alpha = \frac{i - 1}{x + y - 1}$$

43. Ans: 743 nm

Sol: $\frac{1}{355} = \frac{1}{680} - \frac{1}{\lambda}$
 $\lambda = 743 \text{ nm}$

44. Ans: 2-Pentanone



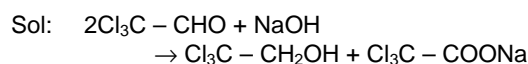
45. Ans: $38.3 \text{ J mol}^{-1} \text{ K}^{-1}$

Sol: $\Delta S = 2.303 nR \log \frac{V_2}{V_1}$
 $= 2.303 \times 2 \times 8.314 \times \log 10$
 $= 38.3 \text{ J K}^{-1}$

46. Ans: Acetaldehyde

Sol: Acetaldehyde reduces Tollen's reagent to metallic silver on warming.

47. Ans: 2, 2, 2-Trichloroethanol



48. Ans: $p(\text{H}_2) = 2 \text{ atm}$ and $[\text{H}^+] = 1.0 \text{ M}$

Sol: $2\text{H}^+ + 2\text{e}^- \rightarrow \text{H}_2$

$$E_{\text{Cl}} = \frac{0.0591}{2} \log \frac{[\text{H}^+]^2}{[\text{H}_2]}$$

$[\text{H}_2] > [\text{H}^+]^2$

49. Ans: 2, 4, 6-Tribromophenol

Sol: Phenol forms 2, 4, 6-tribromophenol when treated with a mixture of KBr , KBrO_3 and HCl .

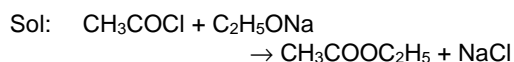
50. Ans: AlCl_3

Sol: Fajan's rules. Al^{3+} is the smallest cation and it has high charge.

51. Ans: BF_6^{3-}

Sol: Boron cannot form BF_6^{3-} since boron has no available d-orbitals.

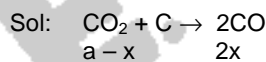
52. Ans: Ethyl ethanoate



53. Ans: Neutral FeCl_3

Sol: Neutral FeCl_3 solution gives violet colour with phenol.

54. Ans: 1.8 atm



$$a = 0.5 \text{ atm}$$

$$a + x = 0.8 \text{ atm}$$

$$x = 0.3 \text{ atm}$$

$$K_p = \frac{p_{\text{CO}}^2}{p_{\text{CO}_2}} = \frac{(0.6)^2}{0.2} = 1.8 \text{ atm}$$

55. Ans: $\text{CH}_3\text{CH}_2\text{CH}(\text{Cl})\text{CO}_2\text{H}$

Sol: Presence of Cl having $-I$ effect on the α -carbon makes 2-chlorobutanoic acid the strongest acid among the given compounds.

56. Ans: $\text{Al}_2\text{O}_3 < \text{MgO} < \text{Na}_2\text{O} < \text{K}_2\text{O}$

Sol: K_2O is more basic than Na_2O . Al_2O_3 is amphoteric.

57. Ans: 0.086

Sol: Mole fraction of methanol
 $= \frac{\text{moles of methanol}}{\text{total moles}} = \frac{5.2}{5.2 + \frac{1000}{18}}$
 $= 0.086$

58. Ans: 2nd

Sol: RNA contains β -D-ribose while DNA contains β -D-2-deoxyribose.

59. Ans: The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table.

Sol: Stability of hydrides decreases from NH_3 to BiH_3 .

60. Ans: The oxidation state of sulphur is never less than +4 in its compounds

Sol: Sulphur exhibits oxidation state lower than +4 in its compounds.

PART – B – PHYSICS

61. Ans: 372 K and 310 K

Sol: $1 - \frac{T_2}{T_1} = \frac{1}{6}$

$$1 - \frac{T_2 - 62}{T_1} = \frac{1}{3}$$

$$\frac{T_2}{T_1} = \frac{5}{6}$$

$$\frac{T_2 - 62}{T_1} = \frac{2}{3}$$

$$\frac{T_2}{T_2 - 62} = \frac{5}{4}$$

$$4T_2 = 5T_2 - 310$$

$$T_2 = 310 \text{ K}$$

$$\Rightarrow T_1 = 372 \text{ K}$$

62. Ans: more than 3 but less than 6.

Sol: $\tau = Fr = 40t - 10t^2$

$$\alpha = \frac{\tau}{I} = 4t - t^2$$

$$\frac{d\omega}{dt} = 4t - t^2 \Rightarrow \omega = 2t^2 - \frac{t^3}{3}$$

(Θ At $t = 0$, $\omega = 0$)

At $t = 6$ s. ω again become zero

$$\omega = \frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3} \Rightarrow \theta = \frac{2t^3}{3} - \frac{t^4}{12}$$

$$\therefore \theta \text{ in } 6 \text{ s} = (144 - 108) = 36 \text{ rad}$$

$$\Rightarrow N = \frac{\theta}{2\pi} = \frac{36}{2\pi} = 5.72 \text{ rotation.}$$

63. Ans: $\frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

Sol: $P_1V = n_1KT_1$

$$P_2V = n_2KT_2$$

$$P_3V = n_3KT_3$$

$$\frac{1}{2}mv^2 = \frac{3}{2}KT_1 \times n_1 + \frac{3}{2}KT_2n_2 + \frac{3}{2}KT_3n_3$$

$$= \frac{3}{2}K(n_1 + n_2 + n_3)T$$

$$T = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$$

64. Ans: 0.15 mV

Sol: $\epsilon = B\lambda v$

$$= 5 \times 10^{-5} \times 2 \times 1.50$$

$$= 0.15 \text{ mV}$$

65. Ans: First increases and then decreases.

Sol: Angular momentum is conserved.

I decreases ω increases then I increases ω decreases.

66. Ans: $v \propto x$

Sol: $T \cos\theta = mg$

$$T \sin\theta = F$$

$$\tan\theta = \frac{F}{mg}$$

$$\frac{x}{2\lambda} = \frac{F}{mg}$$

$$F \propto x$$

$$\int v dv \propto \int x dx$$

$$v^2 \propto x^2$$

$$v \propto x$$

67. Ans: 8.4 kJ

Sol: $\Delta U = mC\Delta T$

$$= 4184 \times 20 \times 0.1$$

$$= 8.4 \text{ kJ}$$

68. Ans: 20 min

Sol: $N = \frac{N_0}{2^{t/T_{1/2}}}$

$$\frac{N_0}{3} = \frac{N_0}{2^{t_2/20}} \Rightarrow t_2 = 20 \frac{\log 3}{\log 2}$$

$$N_0 \frac{2}{3} = \frac{N_0}{2^{t_1/20}} \Rightarrow t_1 = \frac{20(\log 3 - \log 2)}{\log 2}$$

$$t_2 - t_1 = \frac{20}{\log 2} (\log 3 - \log 3 + \log 2) = 20 \text{ min}$$

69. Ans: 108.8 eV

Sol: $\frac{13.6 Z^2}{n^2} = 13.6 \times 9 \left[1 - \frac{1}{9} \right]$

$$= 13.6 \times 9 \times \frac{8}{9}$$

$$= 108.8 \text{ eV}$$

70. Ans: $-6 \epsilon_0 a$

$$= \frac{2 \times 2.5}{2.5} = 2$$

Sol: $V = ar^2 + b$

$$E = -\frac{dV}{dr} = -2ar$$

$$4\pi r^2 \cdot E = \frac{Q}{\epsilon_0}$$

$$Q = -4\pi r^2 \cdot 2ar \cdot \epsilon_0$$

$$\rho = \frac{-8\pi ar^3 \epsilon_0}{\frac{4}{3}\pi r^3}$$

$$= -6 \epsilon_0 a$$

71. Ans: $0.4\pi \text{ mJ}$

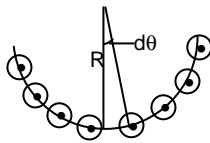
$$\begin{aligned} \text{Sol: } E &= T \cdot 8\pi(r_2^2 - r_1^2) \\ &= 8\pi T \left(\frac{25}{10^4} - \frac{9}{10^4} \right) \\ &= 8 \times 16 \times \pi \times 0.03 \times 10^{-4} \\ &= 0.4\pi \text{ mJ} \end{aligned}$$

72. Ans: $2.7 \times 10^6 \Omega$

$$\begin{aligned} \text{Sol: } V &= V_0(1 - e^{-t/RC}) \\ 120 &= 200(1 - e^{-t/RC}) \\ e^{-t/RC} &= \frac{2}{5} \\ e^{t/RC} &= 2.5 \\ \frac{t}{RC} &= 0.4 \times 2.5 \times 2.303 \\ \Rightarrow R &= 2.7 \times 10^6 \Omega \end{aligned}$$

73. Ans: $\frac{\mu_0 I}{\pi^2 R}$

$$\text{Sol: } B = \frac{I}{\pi R} R d\theta \frac{\mu_0}{2\pi R} \sin \theta$$



$$\begin{aligned} &= \frac{\mu_0 I}{2\pi^2 R} \int_0^{\pi/2} \sin \theta d\theta \\ &= \frac{\mu_0 I}{\pi^2 R} \end{aligned}$$

74. Ans: 2 s

$$\begin{aligned} \text{Sol: } \frac{dv}{dt} &= -2.5\sqrt{v} \\ \frac{dv}{\sqrt{v}} &= -2.5 dt \\ \Rightarrow -2.5t &= \left[2\sqrt{v} \right]_{6.25}^0 \\ t &= \frac{2\sqrt{6.25}}{2.5} \end{aligned}$$

75. Ans: Statement-1 is true, Statement-2 is true and Statement -2 is not the correct explanation of statement - 1

Sol: Statement-1 is true, Statement-2 is true and Statement -2 is not the correct explanation of statement - 1

76. Ans: $\frac{-9Gm}{r}$

$$\begin{aligned} \text{Sol: } \frac{Gm}{x^2} &= \frac{G4m}{(r-x)^2} \\ \frac{(r-x)^2}{x^2} &= 4 \\ r-x &= 2x \\ x &= \frac{r}{3} \\ V &= \frac{-Gm}{r} - \frac{G4m}{\frac{2r}{3}} \\ &= -\frac{Gm}{r} (3+6) \\ &= \frac{-9Gm}{r} \end{aligned}$$

77. Ans: Statement 1 is true. Statement 2 is true. and statement 2 is the correct explanation for statement - 1.

Sol: Statement 1 is true. Statement 2 is true. and statement 2 is the correct explanation for statement - 1.

78. Ans: $\frac{\pi}{4} \sqrt{LC}$

$$\begin{aligned} \text{Sol: } q' &= q_0 \cos \omega t \\ E &= \frac{q_0^2}{2C} \\ \frac{E}{2} &= \frac{1}{2} \frac{q_0^2}{2C} \\ \text{i.e. } q' &= \frac{q_0}{\sqrt{2}} \\ \frac{q_0}{\sqrt{2}} &= q_0 \cos \omega t \\ \Rightarrow \omega t &= \frac{\pi}{4} \\ t &= \frac{\pi}{4} \sqrt{LC} \end{aligned}$$

79. Ans: Statement - 1 is false, Statement-2 is true.

Sol: If $v \Rightarrow 2v$,
 $V_0' > 2V_0$, well known result
 \Rightarrow Statement 1 is wrong.
 Statement 2 is true.

$$\propto (1.001)^2 \lambda^2$$

$$\frac{\Delta R}{R} = 0.002$$

$$\therefore 0.002 \times 100$$

$$= 0.2\%$$

80. Ans: $3.6 \times 10^{-3} \text{ m}$

Sol: $P_0 + \frac{1}{2} \rho v_1^2 + \rho gh$
 $= P_0 + \frac{1}{2} \rho v_2^2$
 $\Rightarrow 2gh = (v_2^2 - v_1^2)$
 $\Rightarrow 2gh + v_1^2 = v_2^2$;
 $v_1 = 0.4 \text{ m s}^{-1}$, $h_2 = 0.2 \text{ m}$
 $\Rightarrow v_2 = 2.0396 \text{ m s}^{-1}$
 $A_1 v_1 = A_2 v_2 \Rightarrow d_2^2 = \frac{d_1^2 v_1}{v_2}$
 $\Rightarrow d_2 = d_1 \sqrt{\frac{v_1}{v_2}}$
 $= 8 \times 10^{-3} \times \sqrt{\frac{0.4}{2.0396}}$
 $\approx 3.6 \times 10^{-3} \text{ m}$

81. Ans: $\left(\frac{M+m}{M}\right)^{1/2}$

Sol: $Mv_1 = (M+m)v_2$
 $\frac{v_1}{v_2} = \frac{M+m}{M}$
 $\frac{1}{2}(M+m)v_2^2 = \frac{1}{2}KA_2^2$
 $\frac{1}{2}Mv_1^2 = \frac{1}{2}KA_1^2$
 $\frac{1}{2}Mv_1^2 = \frac{1}{2}KA_1^2$
 $\Rightarrow \frac{A_1^2}{A_2^2} = \frac{M}{M+m} \left(\frac{M+m}{M}\right)^2$
 $= \frac{M+m}{M}$
 $\therefore \frac{A_1}{A_2} = \left(\frac{M+m}{M}\right)^{1/2}$

82. Ans: $\frac{\pi}{2}$

Sol: Particle 1 is at equilibrium position ($\phi = 0$).
 Particle 2 is at maximum position. $\left(\phi = \frac{\pi}{2}\right)$

83. Ans: Increases by 0.2%

Sol: $R \propto \lambda^2$
 $R' \propto \lambda'^2$

84. Ans: $\frac{\pi v^4}{g^2}$

Sol: $R_{\max} = \frac{v^2}{g}$
 Area = $\pi(R_{\max})^2$
 $= \frac{\pi v^4}{g^2}$

85. Ans: $\frac{1}{2} \frac{Mv^2(\gamma-1)}{R}$

Sol: Volume is constant
 $C_v = \frac{R}{(\gamma-1)}$
 $KE = \frac{1}{2} Mv^2$
 $\Delta Q = nC_v \Delta\theta = 1 \times C_v \Delta\theta$
 $\therefore \Delta\theta = \frac{KE}{C_v} = \frac{1}{2} \frac{Mv^2(\gamma-1)}{R}$

86. Ans: 0.052 cm

Sol: $LC = \frac{1}{100} = 0.01 \text{ mm}$
 Reading = $PSR \times \text{pitch} + CSR \times LC$
 $= 0 + 52 \times 0.01$
 $= 0.52 \text{ mm}$
 $= 0.052 \text{ cm}$

87. Ans: $\frac{2}{3} g$

Sol: $mg - T = ma$
 $TR = \frac{mR^2}{2} \cdot \frac{a}{R}$
 $\Rightarrow mg = \frac{3}{2} ma$
 $\Rightarrow a = \frac{2}{3} g$

88. Ans: Wave moving in $-x$ direction with speed

$$\sqrt{\frac{b}{a}}$$

Sol: $y(x, t) = e^{-(\sqrt{a}x + \sqrt{b}t)^2}$

This is of the form $y(x, t) = f(x + vt)$, where

$v = \frac{\sqrt{b}}{\sqrt{a}}$ travels in negative x direction.

89. Ans: $\frac{1}{15^2} \times 15 = \frac{1}{15} \text{ m s}^{-1}$

Sol: $\frac{1}{v} + \frac{1}{-2.8} = \frac{1}{0.2}$

$$\Rightarrow \frac{1}{v} = \frac{15}{2.8}$$

$$v = \frac{2.8}{15}$$

$$\frac{v}{u} = \frac{1}{15}$$

$$\frac{v^2}{u^2} = \frac{1}{15^2}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{dv}{du} = -\frac{v^2}{u^2}$$

$$\left| \frac{dv}{dt} \right| = \frac{v^2}{u^2} \cdot \frac{du}{dt}$$
$$= \frac{1}{15^2} \times 15 = \frac{1}{15} \text{ m s}^{-1}$$

90. Ans: 45°

Sol: $\mu_1[\hat{N} \times K_1] = \mu_2[\hat{N} \times K_2]$. But plane of separation need to be XY.

