Test Booklet Code - R

AIEEE 2011

Questions and Solutions

PAPER - 1 : MATHEMATICS, CHEMISTRY & PHYSICS

PART- A : MATHEMATICS

1. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval :

(1) $\left(\frac{11}{12}, 1\right]$ (2) $\left(\frac{1}{2}, \frac{3}{4}\right]$ (3) $\left(\frac{3}{4}, \frac{11}{12}\right]$ (4) $\left[0, \frac{1}{2}\right]$

Solution : (4)

P (at least one failure) = 1 – P (all success)

$= 1 - {}^5C_5\, p^5$

Hence, $1 - {}^5C_5 p^5 \geq \frac{31}{32}$

$\Rightarrow \quad p^5 \leq \frac{1}{32}$

$\Rightarrow \quad p \leq \frac{1}{2}$

Hence $p \in \left[0, \frac{1}{2}\right]$

2. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is :

(1) 132 (2) 144 (3) –132 (4) –144

Solution : (4)

$$\left(1 - x - x^2 + x^3\right)^6 = \left(1 - x^2\right)^6 (1 - x)^6$$

$$= \left(C_0 - C_1x^2 + C_2x^4 + C_3x^6 + \ldots + C_6x^{12}\right)\left(C_0 - C_1x + C_2x^2 + \ldots + C_6x^6\right)$$

Coefficient of X^7 is

$$(-C_1)(-C_5) + (C_2)(-C_3) + (-C_3)(-C_1)$$

$$= (-6)\times(-6) + 15\times(-20) + (-20)\times(-6)$$

$= 36 - 300 + 120 = -300 + 156 = -144$

3. $\lim_{x\to 2}\left(\frac{\sqrt{1 - \cos\{2(x-2)\}}}{x - 2}\right)$

(1) equals $\frac{1}{\sqrt{2}}$ (2) does not exist (3) equals $\sqrt{2}$ (4) equals $-\sqrt{2}$

Solution : (2)

$$\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$$

$$= \lim_{x\to 2}\frac{\sqrt{2}\,|\sin(x-2)|}{x-2}$$

$x - 2 = t, \quad x \to 2, \quad t \to 0$

$$= \lim_{t\to 0}\sqrt{2}\,\frac{|\sin t|}{t}$$

$$\text{L.H.L} = \lim_{t\to 0} -\sqrt{2}\,\frac{\sin t}{t} = -\sqrt{2}$$

$$\text{R.H.L} = \lim_{t\to 0} \sqrt{2}\,\frac{\sin t}{t} = \sqrt{2}$$

So L.H.L ≠ R.H.L does not exist.

4. Let R be the set of real numbers.

Statement – 1 :

$A = \{(x, y) \in R \times R : y - x \text{ is an integer}\}$ is an equivalence relation on R.

Statement – 2 :

$B = \{(x, y) \in R \times R : x = \alpha y \text{ for some rational number } \alpha\}$ is an equivalence relation on R.

(1) Statement – 1 is false, Statement – 2 is true.

(2) Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1.

(3) Statement – 1 is true, Statement – 2 is true, Statement – 2 is not a correct explanation for Statement – 1.

(4) Statement – 1 is true, Statement – 2 is false.

Solution : (3)

Statement – 1 :

$A = \{(x, y) \in R \times R : y - x \text{ is integer}\}$

$(x, x) \in R \times R$ as $x - x = 0$ integer ⇒ Reflexive

Also, $(x, y) \in R \times R \Rightarrow x - y = \text{integer} \Rightarrow$ Symmetric

$\Rightarrow y - x = \text{integer}$

Also, $(x, y) \in R \times R$ & $(y, z) \in R \times R$

$\Rightarrow x - y = \text{integer}$ & $y - z = \text{integer}$

So, $x - z = (x - y) + (y - z)$

$= \text{integer} - \text{integer}$

$= \text{integer}$

⇒ transitive

⇒ equivalence relation on R.

Statement – 2 :

Obviously reflexive & symmetric

Let $(x, y) \in R \times R \Rightarrow x = \alpha y$

$(y, z) \in R \times R \Rightarrow y = \beta z$

So, $x = \alpha y$

$= \alpha(\beta z)$

$= (\alpha\beta) z$

⇒ transition

⇒ equivalence relation.

5. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re z = 1, then it is necessary that :

(1) $\beta \in (1, \infty)$ (2) $\beta \in (0, 1)$ (3) $\beta \in (-1, 0)$ (4) $|\beta| = 1$

Solution : (1)

$z^2 + \alpha z - \beta = 0$

$\Rightarrow \quad z = \dfrac{-\alpha \pm \sqrt{\alpha^2 + 4\beta}}{2}$

$= -\dfrac{\alpha}{2} \pm \dfrac{\sqrt{\alpha^2 + 4\beta}}{2}$

Now, $-\dfrac{\alpha}{2} = 1$

$\Rightarrow \quad \alpha = -2$

Now, $\alpha^2 + 4\beta < 0$

$\Rightarrow \quad 4 - 4\beta < 0$

$\Rightarrow \quad -\beta < -1$

$\Rightarrow \quad \beta > 1$

Hence, $\beta \in (1, \infty)$

6. $\dfrac{d^2x}{dy^2}$ equals :

(1) $-\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$

(2) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$

(3) $-\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$

(4) $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-2}$

Solution : (1)

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dx}{dy}\right)^{-1}$$

$$= \frac{d}{dy}\cdot\left(\frac{dx}{dy}\right)^{-1}\cdot\frac{dy}{dx} = -\left(\frac{dx}{dy}\right)^{-2}\cdot\frac{d^2x}{dy^2}\cdot\frac{dy}{dx}$$

$$= -\frac{1}{\left(\frac{dx}{dy}\right)^3}\cdot\frac{d^2x}{dy^2}$$

$$\Rightarrow -\frac{d^2y}{dx^2}\left(\frac{dx}{dy}\right)^3 = \frac{d^2x}{dy^2}$$

$$\Rightarrow \frac{d^2x}{dy^2} = -\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)^{-3}$$

7. The number of values of k for which the linear equations

$4x + ky + 2z = 0$
$kx + 4y + z = 0$
$2x + 2y + z = 0$

possess a non–zero solution is :

(1) zero (2) 3 (3) 2 (1)

Solution : (3)

$$\begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$$\xrightarrow{R_1 - 2R_3} \begin{vmatrix} 0 & k-4 & 0 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $(k - 4)(2 - k) = 0$

$\Rightarrow$ $k = 2, 4$

8. Statement – 1 :

The point A(1, 0, 7) is the mirror image of the point B (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

Statement – 2 :

The line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining A(1, 0, 7) and B(1, 6, 3).

(1) Statement – 1 is false, Statement – 2 is true.
(2) Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1.
(3) Statement – 1 is true, Statement – 2 is true, Statement – 2 is not a correct explanation for Statement – 1.
(4) Statement – 1 is true, Statement – 2 is false.

Solution : (2)

direction of Line AB = (1 – 1, 6 – 0, 3 – t)
= (0, 6, – 4)

So if mirror image $\Rightarrow$ AB and $\frac{k}{1} = \frac{y-2}{2} = \frac{z-7}{3}$ are perpendicular .

So, $a_1 a_2 + b_1 b_2 + c_1 c_2$
$= 0 \times 1 + 6 \times 2 - 4 \times 3$
$= 0$
$\Rightarrow$ perpendicular

Also, Mid point of A and B is (1, 3, 5) has to satisfy

$\Rightarrow \frac{1}{1} = \frac{3-1}{2} = \frac{5-2}{3}$

So, statement (1) correct.

Statement – 2

By definition of mirror image line perpendicular bisector of the joining A (1, 0, 7) and B (1, 6, 3).
So, statement 2 is correct.

9. Consider the following statements

P : Suman is brilliant

Q : Suman is rich
R : Suman is honest
The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as :

(1) $\sim (P \wedge \sim R) \leftrightarrow Q$ (2) $\sim P \wedge (Q \leftrightarrow \sim R)$
(3) $\sim (Q \leftrightarrow (P \wedge \sim R))$ (4) $\sim Q \leftrightarrow \sim P \wedge R$

Solution : (3)

10. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement – 1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

Statement – 2 :
In any triangle, bisector of an angle divides the triangle into two similar triangles.

(1) Statement – 1 is false, Statement – 2 is true.
(2) Statement – 1 is true, Statement – 2 is true; Statement – 2 is a correct explanation for Statement – 1.
(3) Statement – 1 is true, Statement – 2 is true, Statement – 2 is not a correct explanation for Statement – 1.
(4) Statement – 1 is true, Statement – 2 is false.

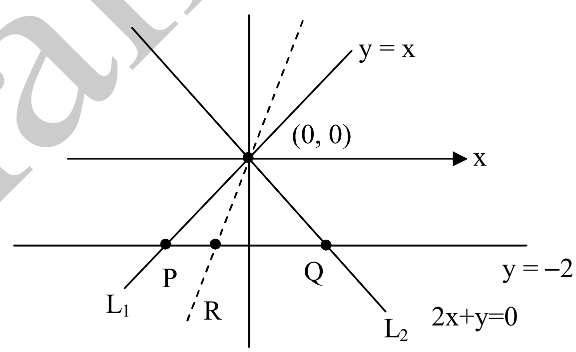
Solution :(4)

Equation of angle bisector

$$\frac{x-y}{\sqrt{2}} = -\frac{2x+y}{\sqrt{5}}$$

$$\left(\sqrt{5}+2\sqrt{2}\right)x + \left(\sqrt{2}-\sqrt{5}\right)y = 0$$

$$P \equiv (-2, -2)\ ,\ Q \equiv (1, -2),\ R \equiv \left(\frac{2(\sqrt{2}-\sqrt{5})}{\sqrt{5}+2\sqrt{2}}, -2\right)$$



$$PR = 2 - \frac{2\left(\sqrt{5}-\sqrt{2}\right)}{\sqrt{5}+2\sqrt{2}}$$

$$= \frac{6\sqrt{2}}{\sqrt{5}+2\sqrt{2}}$$

$$RQ = 1 + \frac{\left(\sqrt{5}-\sqrt{2}\right)2}{\sqrt{5}+2\sqrt{2}} = \frac{3\sqrt{5}}{\sqrt{5}+2\sqrt{2}}$$

$$PR : RQ = 2\sqrt{2} : \sqrt{5}$$

Statement 1 : Is true
Statement 2 : Is false

11. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases Rs. 40 more than the saving of immediate previous month. His total saving from the start of service will be Rs. 11040 after :

(1) 21 months (2) 18 months (3) 19 months (4) 20 months

Solution : (1)

Total saving in (n + 3) months

$$200 + 200 + 200 + 240 + 280 + \ldots\ldots + (200 + 40\,n)$$

$$= 400 + \frac{(200 + 200 + 40\,n)\,(n+1)}{2}$$

$$= 400 + (200 + 20\,n)\,(n + 1)$$

Now,

$400 + (200 + 20n)(n + 1) = 11040$

$\Rightarrow 20(10 + n)(n + 1) = 11040 - 400 = 10640$

$\Rightarrow 10n + n^2 + 10 + n = 532$

$\Rightarrow n^2 + 11n - 522 = 0$

$\Rightarrow n^2 + 29n - 18n - 522 = 0$

$\Rightarrow n(n + 29) - 18(n + 29) = 0$

$\Rightarrow (n + 29)(n - 18) = 0$

$\Rightarrow n = 18$

Hence total months (18 + 3) = 21

12. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is :

(1) $5x^2 + 3y^2 - 32 = 0$
(3) $3x^2 + 5y^2 - 32 = 0$
(3) $5x^2 + 3y^2 - 48 = 0$
(4) $3x^2 + 5y^2 - 15 = 0$

Solution :(2)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Given it passes through (–3, 1)

$\Rightarrow \frac{9}{a^2} + \frac{1}{b^2} = 1$ ……(1)

Also given $e^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{2}{5} = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{b^2}{a^2} = 1 - \frac{2}{5} = \frac{3}{5}$

$\Rightarrow b^2 = \frac{3a^2}{5}$ ……(2)

From (1) & (2),

$\frac{9}{a^2} + \frac{5}{3a^2} = 1 \Rightarrow 9 + \frac{5}{3} = a^2 \Rightarrow a^2 = \frac{32}{3}$

From (2), $b^2 = \frac{3}{5} \times \frac{32}{3} = \frac{32}{5}$

So ellipse is $\frac{x^2}{\frac{32}{3}} + \frac{y^2}{\frac{32}{5}} = 1$

$\Rightarrow 3x^2 + 5y^2 - 32 = 0$

13. If A = $\sin^2 x + \cos^4 x$, then for all real x :

(1) $\frac{3}{4} \le A \le \frac{13}{16}$
(2) $\frac{3}{4} \le A \le 1$
(3) $\frac{13}{16} \le A \le 1$
(4) $1 \le A \le 2$

Solution :(2)

$A = \sin^2 x + \cos^2 x.\ \cos^2 x \le \sin^2 x + \cos^2 x = 1$ … (1)

Also,

$$A = (1 - \cos^2 x) + (\cos^4 x)$$
$$= \cos^4 x - \cos^2 x + 1$$
$$= \left(\cos^2 x - \frac{1}{2}\right)^2 + 1 - \frac{1}{4}$$
$$= \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4} \qquad \ldots (2)$$

from (1) and (2)

$$\frac{3}{4} \leq A \leq 1$$

14. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is :

(1) log 2 (2) π log 2 (3) $\frac{\pi}{8}$ log 2 (4) $\frac{\pi}{2}$ log 2

Solution :(2)

$$I = \int_0^1 \frac{8\log(1+x)}{1+x^2}dx$$

Put x = tan θ

$$I = 8\int_0^{\pi/4} \log(1+\tan\theta)\,d\theta$$
$$I = 8\int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$$
$$I = 8\int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right)d\theta$$
$$I = \pi \log 2$$

15. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane x + 2y + 3z = 4 is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals :

(1) $\frac{5}{3}$ (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) $\frac{2}{5}$

Solution :(2)

$$\cos^{-1}\left(\sqrt{\frac{5}{14}}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{1+4+3\lambda}{\sqrt{5+\lambda^2}\sqrt{14}}\right)$$
$$\Rightarrow \sqrt{1-\frac{5}{14}} = \frac{5+3\lambda}{\sqrt{5+\lambda^2}\sqrt{14}}$$
$$\Rightarrow 3 = \frac{5+3\lambda}{\sqrt{5+\lambda^2}}$$
$$\Rightarrow 45 + 9\lambda^2 = 25 + 9\lambda^2 + 30\lambda$$
$$\frac{2}{3} = \lambda$$

16. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t}\ \sin t\, dt$. Then f has :

(1) local maximum at π and local minimum at 2π.
(2) local maximum at π and 2π.
(3) local minimum at π and 2π.
(4) local minimum at π and local maximum at 2π.

Solution :(1)

$f'(x) = \sqrt{x} sinx$

$f'(x)$ changes its sign from (+) to (–) at x = π and from (–) to (+) at x = 2π.

$\therefore\ f(x)$ has local maximum at π and local minimum at x = 2π.

17. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is **:**

(1) $(-\infty, \infty) - \{0\}$ (2) $(-\infty, \infty)$ (3) $(0, \infty)$ (4) $(-\infty, 0)$

Solution :(4)

$$f(x) \begin{cases} \frac{1}{\sqrt{-2x}} & \text{if } x < 0 \\ \frac{1}{\sqrt{0}} & \text{if } x > 0 \end{cases}$$

$\therefore\ f(x)$ is defined far $x \in (-\infty, 0)$

18. If the mean deviation about the median of the numbers a, 2a, ….., 50a is 50, then $|a|$ equals:

(1) 5 (2) 2 (3) 3 (4) 4

Solution :(4)

Median $= \frac{25a + 26a}{2} = 25.5a$

$\therefore$ Mean deviation $= \frac{|a - 25.5a| + |2a - 25.5a| + ...+ |50a - 25.5a|}{50} = 50$

$\Rightarrow\ 2(|0.5a| + |1.5a| + ...+ |24.5a|) = 2500$

$\Rightarrow\ 2 \times \frac{25}{2}(|a|(25)) = 2500$

$\Rightarrow\ |a| = 4$

19. If $\vec{a} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})$ and $\vec{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$, then the value of $(2\vec{a} - \vec{b}).\left[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})\right]$ is :

(1) 3 (2) –5 (3) –3 (4) 5

Solution : (2)

$|a| = 1,$

$|b| = 1,$

$\vec{a}.\vec{b} = 0,$

$\left(2\vec{a} - \vec{b}\right) . \left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} + 2\vec{b}\right)\right]$,

$= \left(2\vec{a} - \vec{b}\right) . \left(\vec{b} - 2\vec{a}\right)$

$= - |2a - b|^2$

$= - \left(4|a|^2 + |b|^2 - 4\vec{a}.\vec{b}\right)$

$= - 5$

20. The values of p and q for which the function $f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$

is continuous for all x in R, are :

(1) $p = \frac{1}{2}, q = \frac{3}{2}$ (2) $p = \frac{1}{2}, q = -\frac{3}{2}$ (3) $p = \frac{5}{2}, q = \frac{1}{2}$ (4) $p = -\frac{3}{2}, q = \frac{1}{2}$

Solution :(4)

L.H.L $= \lim\limits_{x \to 0^-} (p + 2) = p + 2$

R.H.L $= \lim\limits_{x \to 0^+} \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}}$

$= \lim\limits_{x \to 0^+} \dfrac{x^2}{X^{3/2}\left[\sqrt{x + x^2} + \sqrt{x}\right]}$

$= \lim\limits_{x \to 0^+} \dfrac{1}{\sqrt{1 + x} + 1} = \dfrac{1}{2}$

$f(0) = q$

$p + 2 = q = \dfrac{1}{2}$

$q = \dfrac{1}{2}, \quad P = \dfrac{-3}{2}$

21. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if :

(1) $|a| = 2c$ (2) $2|a| = c$ (3) $|a| = c$ (4) $a = 2c$

Solution :(3)

$x^2 + y^2 = ax$

$x^2 + y^2 = c^2$

$\Rightarrow ax = c^2 \Rightarrow x = \dfrac{c^2}{a}$

$\therefore y^2 = c^2 - \dfrac{c^4}{a^2} = 0$ for one point of intersection

$\Rightarrow a^2c^2 = c^4$

$\Rightarrow |ac| = |c^2|$

$\Rightarrow |a| = |c|$

$\Rightarrow |a| = c \quad (\because c > 0)$

22. Let I be the purchases value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T-t)$, where k > 0 is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is :

(1) e^{-kT} (2) $T^2 - \frac{I}{k}$ (3) $I - \frac{kT^2}{2}$ (4) $I - \frac{k(T-t)^2}{2}$

Solution :(3)

$$\frac{dV(t)}{dt} = -k(T-t)$$

$$V(t) = \frac{k(T-t)^2}{2} + C$$

at t = 0, V(t) = I

$$I = \frac{kT^2}{2} + C \qquad \Rightarrow \quad C = I - \frac{kT^2}{2}$$

$$V(t) = \frac{k(T-t)^2}{2} + I - \frac{kT^2}{2}$$

$$V(T) = I - \frac{kT^2}{2}$$

23. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is :

(1) $P(C|D) = \frac{P(D)}{P(C)}$ (2) $P(C|D) = P(C)$ (3) $P(C|D) \geq P(C)$ (4) $P(C|D) < P(C)$

Solution :(3)

$$P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)} \qquad (\because C \subset D)$$

$$\therefore \quad P\left(\frac{C}{D}\right) = \frac{P(C)}{P(D)} \geq P(C)$$

24. Let A and B be two symmetric matrices of order 3.

Statement – 1 :

A(BA) and (AB)A are symmetric matrices.

Statement–2 :

AB is symmetric matrix if matrix multiplication of A with B is commutative.

(1) Statement–1 is false, Statement–2 is true

(2) Statement–1 is true, Statement –2 is true; Statement–2 is a correct explanation for Statement–1.

(3) Statement –1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is false.

Solution :(3)

S_1 $(A(BA))' = (BA)' A'$

$= A' B' A'$

$= A (BA)$

$((AB)A)' = A'(AB)' = A'B'A'$
$= (AB)A$

S_1 is true.

S_2 $(AB)' = B'A'$
$= BA$
$= AB$ $(\because AB = BA)$

S_2 is correct but not correct explanation.

25. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals:

(1) (−1, 1) (2) (0, 1) (3) (1, 1) (4) (1, 0)

Solution :(3)

$(1+\omega)^7 = A + B\omega$
$\Rightarrow (-\omega^2)^7 = A + B\omega$
$\Rightarrow -\omega^2 = A + B\omega$
$\Rightarrow A + B\omega + \omega^2 = 0$ which is possible for $(A, B) \equiv (1, 1)$

26. Statement-1 :

The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Statement–2 :

The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(1) Statement−1 is false, Statement−2 is true.
(2) Statement−1 is true, Statement−2 is true; Statement−2 is a correct explanation for Statement−1
(3) Statement−1 is true, Statement −2 is true, Statement−2 is not a correct explanation for Statement−1.
(4) Statement−1 is true, Statement−2 is false.

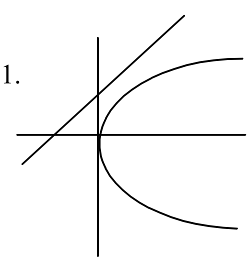
Solution :(3)

27. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is;

(1) $\frac{4}{\sqrt{3}}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\frac{3\sqrt{2}}{8}$ (4) $\frac{8}{3\sqrt{2}}$

Solution : (3)

For a point on the parabola $y^2 = x$ nearest to the line $y - x = 1$,
the normal passing through it will be perpendicular to the line $y - x = 1$.



$\therefore$ (Slope of normal) × 1 $= -1$
$\Rightarrow$ $-t = -1 \Rightarrow t = 1$

Thus, the point is $\left(\frac{1}{4}, \frac{2}{4}\right) \equiv \left(\frac{1}{4}, \frac{1}{2}\right)$

$\therefore$ Shortest distance $= \left|\frac{\frac{1}{4} - \frac{1}{2} + 1}{\sqrt{2}}\right| = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

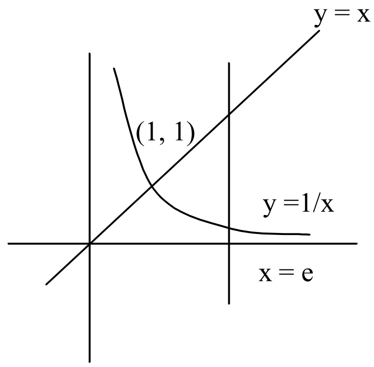
28. The area of the region enclosed by the curves $y = x$, $x = e$, $y = 1/x$ and the positive x−axis is :

(1) $\frac{5}{2}$ square units (2) $\frac{1}{2}$ square units (3) 1 square units (4) $\frac{3}{2}$ square units

Solution : (4)

$$A = \int_0^1 x\,dx + \int_1^e \frac{1}{x}\,dx$$

$$= \frac{1}{2} + 1 = \frac{3}{2}$$



29. If $\frac{dy}{dx} = y + 3 > 0$ and y(0) = 2, then y(ℓn 2) is equal to:

(1) –2 (2) 7 (3) 5 (4) 13

Solution :(2)

$$\frac{dy}{y+3} = dx \Rightarrow \log_e (y+3) = x + c$$

$$\Rightarrow y + 3 = e^{x+c}$$

$$\Rightarrow y + 3 = e^x . e^c$$

$$5 = e^c \quad (\because y(0) = 2)$$

$$\therefore \quad y + 3 = 5e^x$$

$$\therefore \quad y(\log_e 2) = 10 - 3 = 7$$

30. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b}\times\vec{c} = \vec{b}\times\vec{d}$ and $\vec{a}.\vec{d} = 0$. Then the vector $\vec{d}$ is equal to :

(1) $\vec{c} - \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (2) $\vec{b} - \left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$ (3) $\vec{c} + \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (4) $\vec{b} + \left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

Solution :(1)

$$\vec{b}\times\vec{c} = \vec{b}\times\vec{d}$$

$$(\vec{b}\times\vec{c})\times\vec{a} = (\vec{b}\times\vec{d})\times\vec{a}$$

$$(\vec{a}.\vec{b})\vec{c} - (\vec{a}.\vec{c})\vec{b} = (\vec{a}.\vec{b})\vec{d} - (\vec{a}.\vec{d})\vec{b}$$

$$\vec{d} = \vec{c} - \frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\vec{b}$$

PART- B : CHEMISTRY

31. In context of the lanthanoids, which of the following statements is not correct ?
(1) Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series.
(2) There is a gradual decrease in the radii of the members with increasing atomic number in the series.
(3) All the members exhibit +3 oxidation state.
(4) Because of similar properties the separation of lanthanoids is not easy.

Solution : (1)

32. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing from one of the face centred points, the formula of the compound is :
(1) A_2B_5 (2) A_2B (3) AB_2 (4) A_2B_3

Solution : (1)

The atoms on corner contribute $\frac{1}{8}\times 8 = 1$ atom

The atoms on faces contribute $\frac{1}{2}\times 6 = 3$

A_1B_3 should be ideal formula
On removing one face centered atom

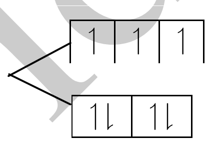
Total contribution by faces $= \frac{1}{2}\times 5 = \frac{5}{2}$

$\therefore A_1B_{5/2} \Rightarrow A_1B_5$

33. The magnetic moment (spin only) of $[NiCl_4]^{2-}$ is :
(1) 1.41 BM (2) 1.82 BM (3) 5.46 BM (4) 2.82 BM

Solution : (4)

$[NiCl_4]^{2-}$
Ni(atomic no.) = 28
$x - 4 = -2 \Rightarrow x = +2$

$Ni^{+2}(26) = 4s^0 3d^8$ 

There are two unpaired electrons for weak ligands,

$\therefore n = 2, \Rightarrow \mu = \sqrt{n(n+2)}$

$= \sqrt{2(2+2)}$

$= \sqrt{8} = 2.82$ B.M.

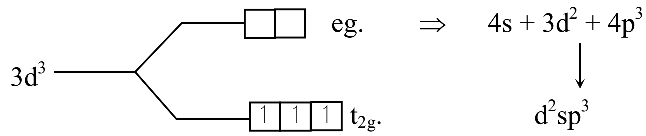
34. Which of the following facts about the complex $[Cr(NH_3)_6]Cl_3$ is wrong ?
(1) The complex gives white precipitate with silver nitrate solution.
(2) The complex involves d^2sp^3 hybridisation and is octahedral in shape.
(3) The complex is paramagnetic.
(4) The complex is an outer orbital complex.

Solution : (4)

$[Cr(NH_3)_6]Cl_3$

(1) $[Cr(NH_3)_6]Cl_3 + 3AgNO_3 \longrightarrow \underset{\text{(White ppt)}}{3AgCl\downarrow} + [Cr(NH_3)_6](NO_3)_3$

(2) $Cr(24) \longrightarrow \overset{+3}{Cr}\,(21) : 4s^0 3d^3$



(3) no. of unpaired electrons n = 3

$\Rightarrow \mu = \sqrt{3(3+2)} = \sqrt{15}$ BM (paramagnetic)

(4) Since hybridization is d^2sp^3, complex is inner octahedral.

35. The rate of a chemical reaction doubles for every 10°C rise of temperature. If the temperature is raised by 50°C , the rate of the reaction increases by about :

(1) 64 times (2) 10 times (3) 24 times (4) 32 times

Solution : (4)

Rate of the reaction increases by 2^5.

36. 'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because

(1) a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6

(2) a and b for Cl_2 > a and b for C_2H_6

(3) a and b for Cl_2 < a and b for C_2H_6

(4) a for Cl_2 < a for C_2H_6 but b for Cl_2 > b for C_2H_6.

Solution : (1)

Greater is the value of 'a' means gas can be easily liquified 'b' depends upon the volume of the gas particles

37. The hybridization of orbitals of N atom in NO_3^- , NO_2^+ and NH_4^+ are respectively :

(1) sp^2 , sp^3 , sp (2) sp, sp^2 , sp^3 (3) sp^2, sp, sp^3 (4) sp, sp^3, sp^2

Solution : (3)

$NO_3^- = sp^2$, $NO_2^+ = sp$, $NH_4^+ = sp^3$

38. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it from freezing at −6°C will be :

(K_f for water = 1.86 K kg mol^{-1} , and molar mass of ethylene glycol = 62 g mol^{-1})

(1) 304.60 g (2) 804.32 g (3) 204.30 g (4) 400.00 g

Solution : (2)

$\Delta T_f = K_f m$

$$6 = 1.86 \times \frac{\omega}{62 \times 4}$$

ω = 800 gm

39. The outer electron configuration of Gd (Atomic No. : 64) is :

(1) $4f^7\, 5d^1\, 6s^2$ (2) $4f^3\, 5d^5\, 6s^2$ (3) $4f^8\, 5d^0\, 6s^2$ (4) $4f^4\, 5d^4\, 6s^2$

Solution : (1)

$$Gd = [Xe]\, 4f^7\, 5d^1\, 6s^2$$

40. The structure of IF_7 is :

(1) pentagonal bipyramid
(2) square pyramid
(3) trigonal bipyramid
(4) octahedral

Solution : (1)

IF_7 is pentagonal bipyramidal.

41. Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of :

(1) an acetylenic triple bond
(2) two ethylenic double bonds
(3) a vinyl group
(4) an isopropyl group

Solution : (3)

$$R - CH = CH_2 \xrightarrow[Zn + H_2O]{O_3} R - CHO + H - \overset{\overset{O}{||}}{C} - H$$

42. The degree of dissociation (α) of a weak electrolyte, A_xB_y is related to van't Hoff factor (i) by the expression :

(1) $\alpha = \dfrac{x+y+1}{i-1}$
(2) $\alpha = \dfrac{i-1}{(x+y-1)}$
(3) $\alpha = \dfrac{i-1}{(x+y+1)}$
(4) $\alpha = \dfrac{x+y-1}{i-1}$

Solution :(2)

$$A_xB_y \longrightarrow xA^{+y} + yB^{-x}$$

	A_xB_y	xA^{+y}	yB^{-x}
t = 0	1	0	0
t = teq	$1 - \alpha$	$x\alpha$	$y\alpha$

$$i = \frac{1 - \alpha + x\alpha + y\alpha}{1}$$

$$\Rightarrow \alpha = \frac{i - 1}{x + y - 1}$$

43. A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emissions is at 680 nm, the other is at :

(1) 518 nm
(2) 1035 nm
(3) 325 nm
(4) 743 nm

Solution : (4)

$$\frac{1}{355} = \frac{1}{680} + \frac{1}{\lambda}$$

$$\lambda = 743\,\text{nm}$$

44. Identify the compound that exhibits tautomerism.

(1) Phenol
(2) 2–Butene
(3) Lactic acid
(4) 2–Pentanone

Solution : (4)

$$H_3C - \overset{\overset{O}{||}}{C} - CH_2 - CH_2 - CH_3 \rightleftharpoons H_3C - \overset{\overset{OH}{|}}{C} = CH - CH_2 - CH_3$$

45. The entropy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of 10 dm^3 to a volume of 100 dm^3 at 27°C is :

(1) 42.3 J mol^{-1} K^{-1} (2) 38.3 J mol^{-1} K^{-1}
(3) 35.8 J mol^{-1} K^{-1} (4) 32.3 J mol^{-1} K^{-1}

Solution :(2)

$$\Delta S = 2.303\ nR\ \log_{10}\frac{V_2}{V_1}$$
$$= 2.303 \times 2 \times 8.314 \times \log_{10}\left(\frac{100}{10}\right)$$
$$= 2.303 \times 2 \times 8.314$$
$$= 38.3\ J\ mol^{-1}\ K^{-1}$$

46. Silver Mirror test is given by which one of the following compounds ?

(1) Benzophenone (2) Acetaldehyde (3) Acetone (4) Formaldehyde

Solution :(2) and (4)

Aldehydes gives positive test with Tollen's Reagent (silver mirror test)

47. Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate and another compound. The other compound is :

(1) Chloroform (2) 2, 2, 2−Trichloroethanol
(3) Trichloromethanol (4) 2, 2, 2−Trichloropropanol

Solution : (2)

$$2CCl_3CHO + NaOH \rightarrow 2CCl_3COONa + \underset{2,2,2-\text{tricholoroethanol}}{CCl_3CH_2OH}$$

48. The reduction potential of hydrogen halfcell will be negative if :

(1) $p(H_2)$ = 2 atm and $[H^+]$ = 2.0 M (2) $p(H_2)$ = 1 atm and $[H^+]$ = 2.0 M
(3) $p(H_2)$ = 1 atm and $[H^+]$ = 1.0 M (4) $p(H_2)$ = 2 atm and $[H^+]$ = 1.0 M

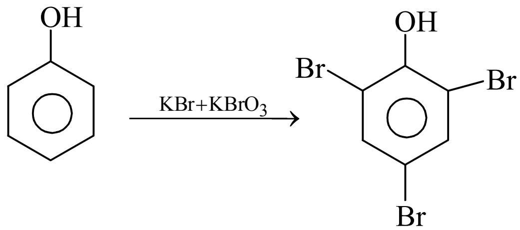
Solution :(4)

$$2H^+ + 2e^- \rightarrow H_2$$
$$E_{RP} = -\frac{0.0591}{2}\log\frac{(P_{H_2})}{[H^+]^2}$$
$$\text{For } 1^{st},\ E_{RP} = -\frac{0.0591}{2}\log\frac{(2)}{(2)^2} = +ve$$
$$\text{For } 2^{nd},\ E_{RP} = -\frac{0.0591}{2}\log_{10}\left(\frac{1}{4}\right) = +ve$$
$$\text{For } 3^{rd},\ E_{RP} = 0$$
$$\text{For } 4^{th},\ E_{RP} = -\frac{-0.0591}{2}\log_{10}\left(\frac{2}{1}\right) = -ve$$

49. Phenol is heated with a solution of mixture of KBr and $KBrO_3$. The major product obtained in the above reaction is :

(1) 2, 4, 6−Tribromophenol (2) 2−Bromophenol
(3) 3−Bromophenol (4) 4−Bromophenol

Solution : (1)



50. Among the following the maximum covalent character is shown by the compound :
(1) $MgCl_2$ (2) $FeCl_2$ (3) $SnCl_2$ (4) $AlCl_3$

Solution :(2)
$MgCl_2$ = Ionic
$AlCl_3$ = Amphoteric
$FeCl_2$ = Covalent

51. Boron cannot form which one of the following anions?
(1) BO_2^- (2) BF_6^{3-} (3) BH_4^- (4) $B(OH)_4^-$

Solution : (2)
Boron doesn't have vacant d–orbital

52. Sodium ethoxide has reacted with ethanoyl chlroride. The compound that is produced in the above reaction is :
(1) Ethyl ethanoate (2) Diethyl ether (3) 2–Butanone (4) Ethyl chloride

Solution : (1)

$$CH_3CH_2ONa + CH_3-\underset{\underset{O}{||}}{C}-Cl \longrightarrow NaCl + CH_3-\underset{\underset{O}{||}}{C}-O-CH_2-CH_3$$

(Ethyl ethanoate)

53. Which of the following reagents may be used to distinguish between phenol and benzoic acid?
(1) Neutral $FeCl_3$ (2) Aqueous NaOH (3) Tollen's reagent (4) Molisch reagent

Solution :(1)

54. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm, the value of K is :
(1) 0.18 atm (2) 1.8 atm (3) 3 atm (4) 0.3 atm

Solution :(2)

$$CO_{2(g)} + C_{(s)} \longrightarrow 2CO_{(g)}$$

t = 0 0.5 atm O
$t = t_{eq}$ (0.5–p) 2p
$P_{Total} = 0.5 - p + 2p = 0.5 + p = 0.8$
$\Rightarrow p = 0.3$ atm

$$kp = \frac{(P_{co})^2}{P_{co2}} = \frac{(2\times0.3)^2}{(0.2)} = \frac{0.36}{0.2} = 1.8\ \text{atm}$$

55. The strongest acid amongst the following compounds is :
(1) $ClCH_2CH_2CH_2COOH$ (2) CH_3COOH
(3) HCOOH (4) $CH_3CH_2CH(Cl)CO_2H$

Solution :(3)

Acidic Nature :

H COOH > CH_3 COOH > $CH_3 - CH_2$ CH (Cl) COOH > Cl CH_2 CH_2 CH_2 COOH

56. Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides ?

(1) $K_2O < Na_2O < Al_2O_3 < MgO$ (2) $Al_2O_3 < MgO < Na_2O < K_2O$
(3) $MgO < K_2O < Al_2O_3 < Na_2O$ (4) $Na_2O < K_2O < MgO < Al_2O_3$

Solution :(2)

Basic nature of oxide : $Al_2O_3 < MgO < Na_2O < K_2O$

57. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH, is supplied. What is the mole fraction of methyl alcohol in the solution ?

(1) 0.050 (2) 0.100 (3) 0.190 (4) 0.086

Solution :(4)

$$\text{Molality} = \frac{X_{CH_3OH} \times 1000}{X_{H_20} \times M_{H_2O}}$$

$$5.2 = \frac{X_{CH_3OH} \times 1000}{\left(1 - X_{CH_3OH}\right) \times 18}$$

$$\frac{x}{(1 - X_{CH_3OH})} = \frac{5.2 \times 18}{1000} = 0.0936$$

$$X_{CH_3OH} = \frac{0.0936}{1.0936} = 0.086$$

58. The pressure or absence of hydroxy group on which carbon atom of sugar differentiates RNA and DNA ?

(1) 4^{th} (2) 1^{st} (3) 2^{nd} (4) 3^{rd}

Solution :(3)

At 2^{nd} position

59. Which of the following statement is wrong?

(1) N_2O_4 has two resonance structures.
(2) The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table.
(3) Nitrogen cannot form $d\pi - p\pi$ bond.
(4) Single N − N bond is weaker than the single P − P bond.

Solution :(2)

Thermal stability decreases gradually from NH_3 to BiH_3.

60. Which of the following statements regarding sulphur is incorrect?

(1) The oxidation state of sulphur is never less than +4 in its compounds.
(2) S_2 molecule is paramagnetic.
(3) The vapour at 200^{o}C consists mostly of S_8 rings.
(4) At 200^{o}C the gas mainly consists of S_2 molecules.

Solution :(1)

Sulphur have −2 oxidation state in H_2S.

PART- C : PHYSICS

61. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively.

(1) 310 K and 248 K
(2) 372 K and 310 K
(3) 372 K and 330 K
(4) 330 K and 268 K

Solution :(2)

$$\eta = 1 - \frac{T_2}{T_1}$$

i.e. $\frac{1}{6} = 1 - \frac{T_2}{T_1}$ … (1)

then $\frac{1}{3} = 1 - \frac{T_2 - 62}{T_1}$ … (2)

$\Rightarrow \frac{1}{3} = 1 - \frac{T_2}{T_1} + \frac{62}{T_1} \qquad = \frac{1}{6} + \frac{62}{T_1}$

$\Rightarrow \frac{62}{T_1} = \frac{1}{6}$

$\therefore \quad T_1 = 372$ K

Putting in eq. (1)

$T_2 = 310$ K

62. A pulley of radius 2 m is rotated about its axis by a force F = $(20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion if reversed, is :

(1) more than 9
(2) less than 3
(3) more than 3 but less than 6
(4) more than 6 but less than 9

Solution :(3)

$\tau = F.R = I\alpha$

$\alpha = \frac{(20t - 5t^2)2}{10} = 4t - t^2$

$\alpha = \frac{d\omega}{dt} = 4t - t^2$

$\omega = \int (4t - t^2)dt$

$= 4\left(\frac{t^2}{2}\right) - \frac{t^3}{3}$

$\omega = 2t^2 - \frac{t^3}{3}$

Then direction will be reversed if, $\omega = 0$

$2t^2 - \frac{t^3}{3} = 0$

$6t^2 - t3 = 0$

$t^2 (6 - t) = 0$

$t = 0$ & $t = 6$ sec

$\omega = \frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$

$\theta = \int\left(2t^2 - \frac{t^3}{3}\right) dt$

$\theta = 2\left(\frac{t^3}{3}\right) - \frac{t^4}{12}$

$\theta = \frac{2t^3}{3} - \frac{t^4}{12}$

after t = 6 sec, angle covered will be –

$\theta = \frac{2}{3}(6)^3 - \frac{(6)^4}{12}$

$= \frac{2}{3} \times 216 - \frac{36 \times 36}{12}$

$\theta = 36$ radians

2π rad. = 6.28 rad. $\rightarrow$ 1 rotation

$\therefore$ 36 radians $\rightarrow \frac{36}{6.28}$ rotations

= 5.73 rotations = 5 rotations

$\therefore$ more than 3 but less than 6.

63. Three perfect gases as absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is :

(1) $\frac{n_1^2T_1^2 + n_2^2T_2^2 + n_3^2T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$ (2) $\frac{(T_1 + T_2 + T_3)}{3}$

(3) $\frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$ (4) $\frac{n_1T_1^2 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$

Solution :(3)

$U = U_1 + U_2 + U_3$

$U_1 = n_1 RT_1$

$U_2 = n_2 RT_2$

$U_3 = n_3 RT_3$

On mixing the gases

$n = n_1 + n_2 + n_3$

$\therefore U = (n_1 + n_2 + n_3) RT$

$\therefore (n_1 + n_2 + n_3) RT = n_1 RT_1 + n_2 RT_2 + n_3 RT_3$

$T = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

64. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} $NA^{-1}m^{-1}$ due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is :

(1) 0.15 mV (2) 1 mV (3) 0.75 mV (4) 0.50 mV

Solution :(1)

$B = 5 \times 10^{-5}$ $NA^{-1}m^{-1}$

$l = 2$ m

$v = 1.5$ m/s

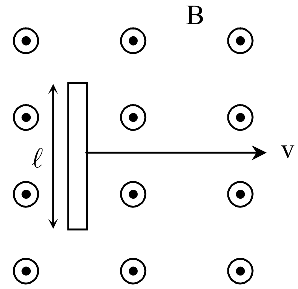
emf induced in the aerial

$e = Blv$

$= 5 \times 10^{-5} \times 2 \times 1.5$

$= 15 \times 10^{-5}$

$e = 0.15$ mV



65. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc :

(1) first increases and then decreases (2) remains unchanged

(3) continuously decreases (4) continuously increases

Solution : (1)

Angular momentum of system is conserved

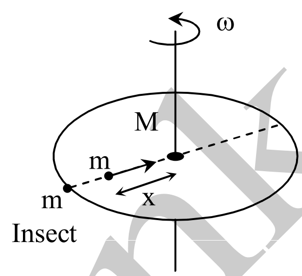
$I\omega = I'\omega'$

$$\left(\frac{1}{2}MR^2 + mR^2\right)\omega = \left(\frac{1}{2}mR^2 + mx^2\right)\omega'$$

$x < R$

$\therefore \quad I' < I$

$\therefore \quad \omega' < \omega$



Initially x will decrease from R to O then increases 0 to R.

$\therefore$ I first decreases & ω increases then I increases & ω decreases.

66. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance d ($d << l$) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them.

(1) $v \propto x$ (2) $v \propto x^{-1/2}$ (3) $v \propto x^{-1}$ (4) $v \propto x^{1/2}$

Solution : (2)

$mg = T\cos\theta$ … (1)

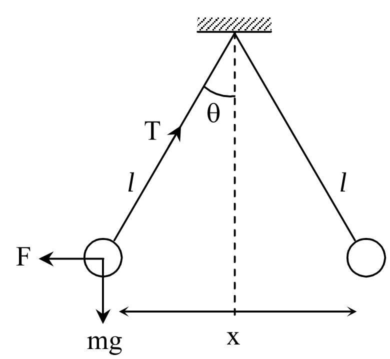
$F = T\sin\theta$ … (2)

$$\frac{F}{mg} = \tan\theta$$

$$F = \frac{q^2}{4\pi \in_0 x^2}$$

and $$\tan\theta = \frac{x/2}{\sqrt{l^2 - (x/2)^2}}$$

$$\tan\theta = \frac{x}{2l} \qquad \text{as } l >> x$$



$$\therefore \quad \frac{q^2}{4\pi \in_0 x^2 mg} = \frac{x}{2l}$$

$$q^2 = \frac{2\pi \in_0 mg}{l} x^3$$

$$q = \sqrt{\frac{2\pi \in_0 mg}{l}}\, x^{3/2}$$

$$\frac{dq}{dt} = \sqrt{\frac{2\pi\epsilon_0 mg}{l}}\ \frac{3}{2}x^{1/2}\ \frac{dx}{dt} \qquad \left[\because \frac{dx}{dt} = v\right]$$

$$x^{1/2}\ v = \frac{2/3}{\sqrt{\frac{2\pi\epsilon_0 mg}{l}}}\ \frac{dq}{dt} \qquad \text{[RHS constant]}$$

$$x^{1/2}\ v = \text{constant}$$

$$v \propto \frac{1}{x^{1/2}}$$

$$v \propto x^{-1/2}$$

67. 100 g of water is heated from 30 °C to 50 °C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K) :
(1) 2.1 kJ (2) 4.2 kJ (3) 8.4 kJ (4) 84 kJ

Solution : (3)

$$Q = mC\ 4T$$
$$= 100 \times 1\ (50 - 30)$$
$$= 200 \text{ Cals.}$$
$$= 8400 \text{ J}$$

So increase in internal energy = 8.4 kJ

68. The half life of a radioactive substance is 20 minutes. The approximate time interval ($t_2 - t_1$) between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is :
(1) 28 min. (2) 7 min (3) 14 min. (4) 20 min.

Solution : (4)

$$\frac{N}{N_0} = \frac{1}{2^n}$$

$$\therefore \quad \frac{1}{3} = \frac{1}{2^{t_2/20}} \qquad \ldots (1)$$

$$\Rightarrow \quad 2^{t_2/20} = 3$$

$$\text{and} \quad \frac{2}{3} = \frac{1}{2^{t_1/20}} \qquad \ldots (2)$$

$$\Rightarrow \quad 2^{t_1/20} = \frac{3}{2}$$

$$\text{dividing} \quad \frac{2^{t_2/20}}{2^{t_1/20}} = 2$$

$$\therefore \quad \frac{t_2}{20} - \frac{t_1}{20} = 1$$

$$\therefore \quad t_2 - t_1 = 20 \text{ min.}$$

69. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is :
(1) 122.4 eV (2) 12.1 eV (3) 36.3 eV (4) 108.8 eV

Solution : (4)

$$En = \frac{E_1 z^2}{n^2}$$

Energy required $= E_3 - E_1$

$$= -\frac{13.6(3)^2}{3^2} - \left(-\frac{13.6 \times 3^2}{1^2}\right)$$

$= -13.6 + 122.4$

$= 108.8$ ev.

70. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is :

(1) $-6\ a\varepsilon_0$ (2) $-24\pi\ a\varepsilon_0 r$ (3) $-6\ a\varepsilon_0 r$ (4) $-24\pi\ a\varepsilon_0$

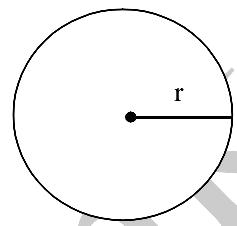
Solution : (1)

$$E = -\frac{d\phi}{dt} = \frac{-d}{dt}(ar^2 + b)$$

$E = -2ar$

Using gauss law –

$$\int \vec{E}.\vec{A} = \frac{q}{\varepsilon_0}$$



$$(-2ar)\ 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$q = -8\pi\varepsilon_0\ a\ r^3$

$$q = \int_0^r \sigma \quad 4\pi r^2 d\,r = -8\pi\varepsilon_0\ ar^3$$

$$\sigma\left[4\pi\frac{r^3}{3}\right]_0^r = -8\pi a\varepsilon_0\ r^3$$

$$\sigma \times \frac{4\pi r^3}{3} = 8\pi\ a\ \varepsilon_0\ r^3$$

(1) $\sigma = -6\ a\ \varepsilon_0$

71. Work done in increasing the size of a soap bubble from the radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = $0.03\ Nm^{-1}$) :

(1) 0.4π mJ (2) 4π mJ (3) 0.2π mJ (4) 2π mJ

Solution : (1)

$W = T\ .\ \Delta\ A$

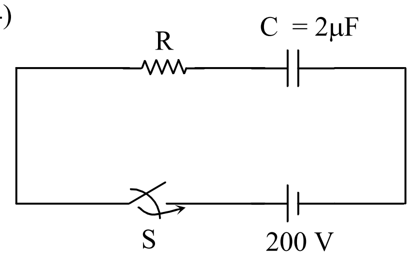
$\therefore$ $W = 0.03 \times 4\pi\ (5^2 - 3^2) \times 10^{-4} \times 2$ (soap bubble has two layers)

$W = 0.4\pi$ mJ

72. A resistor 'R' and $2\mu F$ capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. ($\log_{10} 2.5 = 0.4$)

(1) $3.3 \times 10^7\ \Omega$ (2) $1.3 \times 10^4\ \Omega$ (3) $1.7 \times 10^5\ \Omega$ (4) $2.7 \times 10^6\ \Omega$

Solution : (4)



$$V = V_0\left(1-e^{-t/RC}\right)$$

$$120 = 200\left(1-e^{-\frac{5}{R\times 2\times 10^{-6}}}\right)$$

$$\frac{3}{5} = 1 - e^{-\frac{2.5\times 10^6}{R}}$$

$$e^{-\frac{2.5\times 10^6}{R}} = \frac{2}{5}$$

$$e^{\frac{2.5\times 10^6}{R}} = 2.5$$

$$\frac{2.5\times 10^6}{R} = \ell n 2.5 = 0.4\times 2.303$$

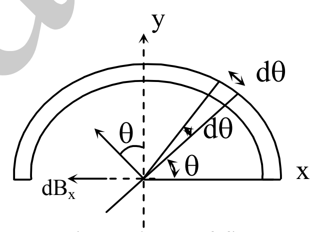
$$R = \frac{2.5\times 10^6}{2.303\times 0.4}$$

$$R = 2.7\times 10^6\,\Omega$$

73. A current I flows in an infinitely long wire with cross section in the form of a semi-circular ring of radius R. The magnitude of the magnetic induction along its axis is :

(1) $\frac{\mu_0 I}{4\pi R}$ (2) $\frac{\mu_0 I}{\pi^2 R}$ (3) $\frac{\mu_0 I}{2\pi^2 R}$ (4) $\frac{\mu_0 I}{2\pi R}$

Solution : (2)



current into plane of figure.

$$di = \left(\frac{d\theta}{\pi}\right)i$$

$$\therefore\ dB = \frac{\mu_0(di)}{2\pi R} = \frac{\mu_0 i}{2\pi^2 R}.d\theta$$

Only dB_x will add.

$$\therefore\ \text{Net induction} = \int_0^\pi dB_x = \frac{\mu_0 i}{2\pi^2 R}\int_0^\pi \sin\theta.d\theta$$

$$\Rightarrow\ B = \frac{\mu_0 i}{\pi^2 R}$$

74. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by :

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be :

(1) 8 s (2) 1 s (3) 2 s (4) 4 s

Solution : (3)

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

$$\frac{dv}{\sqrt{v}} = -2.5\,dt$$

$$\int_{6.25}^{0} v^{-1/2}\,dv = -2.5\int_0^t dt$$

$$2\sqrt{v}\Big|_{6.25}^{0} = -2.5t$$

$$2\left(0-\sqrt{6.25}\right) = -2.5t$$

$$\therefore\ t = 2\text{ s}$$

75. **Direction :** The question has a paragraph followed by two statements, **Statement–1** and **Statement–2**. Of the given four alternatives after the statements, choose the one that describes the statements.
A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement–1 : When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.
Statement-2 : The centre of the interface pattern is dark.

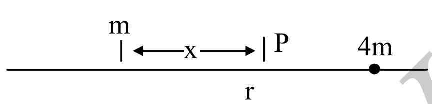
(1) Statement–1 is false, Statement–2 is true.
(2) Statement–1 is true, Statement–2 is false.
(3) Statement–1 is true, Statement–2 is true and Statement–2 is the correct explanation of statement–1.
(4) Statement–1 is true, Statement–2 is true and Statement–2 is not the correct explanation of Statement–1

Solution : (2)
The central fringe is the Newton's ring experiment is bright.

76. Two bodies of masses m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is :

(1) $-\frac{9Gm}{r}$ (2) zero (3) $-\frac{4Gm}{r}$ (4) $-\frac{6Gm}{r}$

Solution : (1)



Let the point be at distance x

$$\frac{Gm}{x^2} = \frac{6.4m}{(r-x)^2} \qquad [\because \text{ Electric field is zero}]$$

$$\Rightarrow\ (r-x)^2 = 4x^2$$

$$\Rightarrow r - x = 2x \Rightarrow x = \frac{r}{3}$$

$$\text{Potential at P} = -\frac{Gm}{\left(\frac{r}{3}\right)} - \frac{G4m}{2\frac{r}{3}}$$

$$= -\frac{3Gm}{r} - \frac{6Gm}{r} = -\frac{9Gm}{r}$$

77. This question has Statement–1 and Statement–2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement–1 : Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.
Statement-2 : The state of ionosphere varies from hour to hour, day to day and season to season.

(1) Statement–1 is false, Statement–2 is true.
(2) Statement–1 is true, Statement–2 is false.
(3) Statement–1 is true, Statement–2 is true, Statement–2 is the correct explanation of Statement–1.
(4) Statement–1 is true, Statement–2 is true, Statement–2 is not the correct explanation of Statement–1

Solution : (1)

78. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic fields is :

(1) $\sqrt{LC}$ (2) $\pi\sqrt{LC}$ (3) $\frac{\pi}{4}\sqrt{LC}$ (4) $2\pi\sqrt{LC}$

Solution : (3)

Charge on capacitor as function of time

$$q = q_0 \cos(\omega t)$$

$$\therefore\ i = -q_0\omega \sin(\omega t)$$

$$\frac{q^2}{2c} = \frac{1}{2}Li^2 \Rightarrow \frac{q_0^2 \cos^2(\omega t)}{2c} = \frac{1}{2} L\, q_0^2\, \omega^2 \sin^2(\omega t)$$

$$\Rightarrow \tan^2(\omega t) = \frac{1}{LC}\cdot\frac{1}{\omega^2} = 1$$

$$\Rightarrow \tan(\omega t) = 1$$

$$\omega t = \frac{\pi}{4} \Rightarrow t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{Lc}$$

79. This question has Statement–1 and Statement–2. Of the four choice given after the statements, choose the one that best describes the two statements.

Statement-1 : A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement-2 : The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(1) Statement–1 is false, Statement–2 is true.
(2) Statement–1 is true, Statement–2 is false.
(3) Statement–1 is true, Statement–2 is true, Statement–2 is the correct explanation of Statement–1.
(4) Statement–1 is true, Statement–2 is true, Statement–2 is not the correct explanation of Statement–1.

(1) $\frac{23}{\sqrt{17}}$ (2) $\frac{23}{\sqrt{15}}$ (3) $\sqrt{17}$ (4) $\frac{17}{\sqrt{15}}$

Solution : (1)

$$KE_{max} = hv - \phi = eV_0$$

$$\Rightarrow V_0 = \frac{hv}{e} - \frac{\phi}{e}$$

$$\therefore\ k_m = hv - \phi \qquad k_m^1 = 2hv - \phi$$

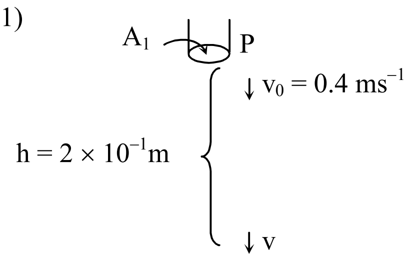
$$V_0 = \frac{hv}{e} - \frac{\phi}{e} \qquad V_0^1 = \frac{2hv}{e} - \frac{\phi}{e}$$

Clearly $k_m^1 \neq 2\,k_m$ and $V_0^1 \neq 2\,V_0^1$.

80. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is $0.4\ ms^{-1}$. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to:

(1) 3.6×10^{-3} m (2) 5.0×10^{-3} m (3) 7.5×10^{-3} m (4) 9.6×10^{-3} m

Solution : (1)



Apply Bernanlli's Theorem,

$$\frac{1}{2}\rho v_0^2 + \rho gh = \frac{1}{2}\rho v^2$$

$$\Rightarrow v = \sqrt{v_0^2 + 2gh} = \sqrt{\left[0.16 + 2\times g\times 2\times 10^{-1}\right]} \simeq 2ms^{-1}$$

$$v_0 r_0^2 = vr^2 \text{ (Equation of continuity)}$$

$$\Rightarrow d = d_0\sqrt{\frac{v_o}{V}} = 8\times 10^{-3}\sqrt{\frac{0.4}{2}}$$

$$\Rightarrow d = 8\times 10^{-3}\times\sqrt{0.2}$$
$$= 8\times 0.45\times 10^{-3}$$
$$= 3.60\times 10^{-3}$$

81. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is :

(1) $\left(\frac{M+m}{M}\right)^{1/2}$ (2) $\frac{M}{M+m}$ (3) $\frac{M+m}{M}$ (4) $\left(\frac{M}{M+m}\right)^{1/2}$

Solution : (1)

$$v_0\left(\sqrt{\frac{K}{M}}\right)A_1$$

After the mass m is dropped the

$$\text{New velocity} = \frac{M}{M+m}\sqrt{\frac{K}{M}}.A_1$$

$$\therefore \frac{1}{2}(M+m)\times\frac{M^2}{(M+m)^2}.\frac{K}{M}.A_1^2 = \frac{1}{2}K.A_2^2 \quad = A_2 = A_1\sqrt{\frac{M}{M+m}}$$

82. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x–axis. Their mean position is separated by distance $X_0(X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Solution : (3)

$$x_1 = A\sin(\omega t + \phi_1)$$

$$x_2 = A\sin(\omega t + \phi_2)$$

$$x_1 - x_2 = A\left[2\sin\left[\omega t + \frac{\phi_1+\phi_2}{2}\right]\sin\left[\frac{\phi_1-\phi_2}{2}\right]\right]$$

$$A = 2A\sin\left(\frac{\phi_1-\phi_2}{2}\right)$$

$$\frac{\phi_1-\phi_2}{2} = \frac{\pi}{6}$$

$$\therefore \phi_1-\phi_2 = \frac{\pi}{3}$$

83. If a wire is stretched to make it 0.1% longer, its resistance will :

(1) decrease by 0.05% (2) increase by 0.05%

(3) increase by 0.2% (4) decrease by 0.2%

Solution : (3)

The volume remains constant when stretched

$$A\ell = \text{constant}$$

$$dA\,\ell + A\,d\ell = 0$$

$$\frac{dA}{A} + \frac{d\ell}{\ell} = 0$$

$$\%A + \%\ell = 0$$

$$\%A = -0.1$$

i.e. The area decreases by 0.1%.

$$R = \frac{\rho\ell}{A}$$

$$dR = \rho\frac{(A\,d\ell - \ell\,dA)}{A^2}$$

$$\frac{dR}{R} = \frac{\rho(A\,d\ell - \ell\,dA)/A^2}{\rho\ell/A} = \frac{d\ell}{\ell} - \frac{dA}{A}$$

$$\therefore\ \%\,R = \%\ell - \%A = 0.1 - (-0.1) = 0.2$$

84. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is :

(1) $\pi\frac{v^2}{g^2}$ (2) $\pi\frac{v^2}{g}$ (3) $\pi\frac{v^4}{g^2}$ (4) $\frac{\pi}{2}\frac{v^4}{g^2}$

Solution : (3)

The maximum range

$$R = \frac{u^2}{g} = \frac{v^2}{g}$$

$$\text{The area} = \pi R^2 = \frac{\pi v^4}{g^2}$$

85. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by :

(1) $\frac{(\gamma-1)}{2R}Mv^2\,K$ (2) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2\,K$

(3) $\frac{(\gamma-1)}{2\gamma R}Mv^2\,K$ (4) $\frac{\gamma Mv^2}{2R}K$

Solution : (1)

$$\Delta Q = \Delta U + \Delta W = 0$$

$$\Delta U = \Delta W = P\Delta V = nC_v\Delta T$$

$$C_p - C_v = R$$

$$C_v\gamma - C_v = R$$

$$C_v(\gamma - 1) = R$$

$$C_v = \frac{R}{\gamma - 1}$$

$$\therefore\ nC_v\Delta T = \left(\frac{1}{2}mv^2\right)$$

$$n\frac{R}{\gamma-1}\Delta T = \frac{1}{2}(nM)v^2$$

$$\Delta T = \frac{(\gamma-1)}{2R}Mv^2 \text{ kelvin}$$

86. A screw gauge gives the following reading when used to measure the diameter of a wire.
Main scale reading : 0 mm.
Circular scale reading : 52 divisions
Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.
The diameter of wire from the above data is :
(1) 0.005 cm (2) 0.52 cm (3) 0.052 cm (4) 0.026 cm

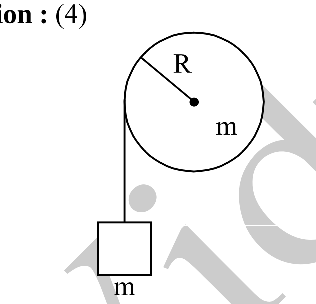
Solution : (3)

1 mm $\rightarrow$ 100 divisions
0.52 mm $\rightarrow$ 52 divisions
$\therefore$ Diameter $= 0.52 \times 10^{-1}$ cm
$= 0.052$ cm.

87. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is :

(1) $\frac{g}{3}$ (2) $\frac{3}{2}g$ (3) g (4) $\frac{2}{3}g$

Solution : (4)



For pulley
$T \times R = I_{c.m.}\,\alpha$

for block

T (↑)
$mg - T = ma$
mg (↓)

$a = R\alpha$

$mg - T = mR\alpha$ … (1)

$$T \times R = \frac{mR^2}{2}\alpha$$

$$\Rightarrow \quad T = \frac{mR\alpha}{2} \quad \ldots (2)$$

$$\text{Adding } mg = \frac{3}{2}mR\alpha$$

$$R\alpha = \frac{2g}{3}$$
$$a = \frac{2g}{3}$$

88. The transverse displacement y(x, t) of a wave on a string is given by

$y(x,t) = e^{-\left(ax^2 + bt^2 + 2\sqrt{ab}\ xt\right)}$. This represents a :

(1) standing wave of frequency $\frac{1}{\sqrt{b}}$

(2) wave moving in +x direction with speed $\sqrt{\frac{a}{b}}$

(3) wave moving in −x direction with speed $\sqrt{\frac{b}{a}}$

(4) standing wave of frequency $\sqrt{b}$

Solution : (3)

$$y\,(x,\,t) = e^{-(ax^2+bt^2+2\sqrt{ab}xt)}$$
$$= e^{-(\sqrt{a}x+\sqrt{b}t)^2}$$

Compare $\sqrt{a}x = \sqrt{b}t$ with kx + wt

$\therefore$ Velocity $= \frac{w}{k}$

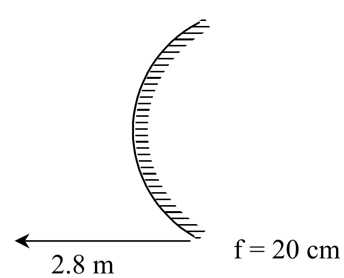
$$= \frac{\sqrt{b}}{\sqrt{a}} \qquad = \sqrt{\frac{b}{a}}$$

Moving leftwards.

89. A car is fitted with a convex side−view mirror of focal length 20 cm. A second car 2.8m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is :

(1) 15 m/s (2) $\frac{1}{10}$ m/s (3) $\frac{1}{15}$ m/s (4) 10 m/s

Solution : (3)



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Differentiating w.r.t. time

$$-\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

$$\frac{du}{dt} = -\,15 \text{ m/s} \qquad [\because \text{ u is decreasing}]$$

$$\frac{dv}{dt} = \frac{v^2}{u^2}\times\left(-\frac{du}{dt}\right)$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{-2.8} = \frac{1}{20\times10^{-2}}$$

$$\frac{1}{v} = \frac{1}{0.2} + \frac{1}{2.8}$$

$$\frac{1}{v} = \frac{3}{0.56}$$

$$v = \frac{56}{300}$$

$$\frac{dv}{dt} = \left(\frac{v}{u}\right)^2 \left(\frac{-du}{dt}\right) = \left(\frac{56/300}{2.8}\right)^2 \times 15 = \left(\frac{20}{300}\right)^2 \times 15$$

$$\frac{dv}{dt} = \frac{1}{15^2} \times 15 = \frac{1}{15} \text{ m/s}$$

90. Let the x − z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\ \hat{i} + 8\sqrt{3}\ \hat{j} - 10\ \hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is :

(1) 75° (2) 30° (3) 45° (4) 60°

Solution : (3)

Let x − y plane be the boundary.

Then the angle of incidence is the angle it makes with the z axis.

$$\vec{A}.\widehat{K} = |A| \cos\theta$$

$$\sqrt{36\times 3 + 64\times 3 + 100}\ \cos\theta = -10$$

$$\sqrt{300 + 100}\ \cos\theta = -10$$

$$\cos\theta = \frac{-10}{20} = \frac{-1}{2}$$

$$\theta = 120^o$$

We take the smaller angle.

$$\therefore \theta = 60^o$$

$$\mu_1 sin\theta_1 = \mu_2 sin\theta_2$$

$$\sqrt{2} \times \frac{\sqrt{3}}{2} = \sqrt{3} \times \sin\theta_2$$

$$\sin\theta_2 = \frac{1}{\sqrt{2}}$$

$$\theta_2 = \frac{\pi}{4}$$

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