SOLUTIONS & ANSWERS FOR KERALA ENGINEERING ENTRANCE EXAMINATION-2011 – PAPER II VERSION – B1

[MATHEMATICS]

1. Ans: $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

Sol: $y = 2^{x(x-1)}$
$\log_2 y = x(x-1)$
$\Rightarrow x^2 - x - \log_2 y = 0$
$x = \frac{1 \pm \sqrt{1+4\log_2 y}}{2}$
$\therefore f^{-1}(x) = \frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

2. Ans: 6

Sol: $n\left((A \cap B)' \cap A\right) = n\left(A' \cup B' \cap A\right)$
$= n(A' \cap A) + n(B' \cap A)$
$= n(A - B) = 8 - 2$
$= 6$

3. Ans: $\sin x^2 + \cos x^2$

Sol: $f(x) = \sin x + \cos x$; $g(x) = x^2$
$fog(x) = \sin x^2 + \cos x^2$.

4. Ans: 32

Sol: $n(A) = 5$
$n(P(A)) = 2^5 = 32$

5. Ans: $-3 < x < 3$

Sol: $f(x) = \frac{1}{\sqrt{9-x^2}}$
$\Rightarrow 9 - x^2 > 0$
$\therefore -3 < x < 3$

6. Ans: $\frac{\pi}{2}$.

Sol: Period of the function $|\sin 2x| + |\cos x|$

Period of $|\sin 2x| = \frac{\pi}{2}$; Period of $|\cos 8x| = \frac{\pi}{4}$

$\therefore$ The required answer is $\frac{\pi}{2}$.

7. Ans: 0

Sol: $i^1 - i^2 + i^3 - i^4 + \ldots.. i^{100}$
This is a GP with $a = i$ and $r = -i$
$S_{100} = \frac{i\left(1-i^{100}\right)}{1+i} = 0$

8. Ans: $-\frac{1}{2}$

Sol: $\ln\left(\frac{2+i}{ai-1} \times \frac{(-1-ai)}{(-1-ai)}\right)$
$= \frac{-2a-1}{1+a^2} = 0$
$\therefore a = -\frac{1}{2}$

9. Ans: $\frac{\pi}{2}$.

Sol: $\left(\frac{i}{2} - \frac{2}{i}\right) = \frac{i}{2} + 2i = \frac{5i}{2}$
$\therefore \arg\left(\frac{i}{2} - \frac{2}{i}\right) = \frac{\pi}{2}$

10. Ans: $-|z_1 - z_2|^2$

Sol: $Z_1 = 3 + 4i$; $Z_2 = -1 + 2i$
$|z_1 + z_2|^2 - 2\left(|z_1|^2 + |z_2|^2\right)$
$= |2 + 6i|^2 - 2(25 + 5)$
$= 40 - 60 = -20$
From the options we get
$-|z_1 - z_2|^2 = -20$

11. Ans: 0

Sol: $(z_1 + z_2) \le |z_1| + |z_2|$
Equality holds when z_1, z_2, $z_1 + z_2$ are collinear with the origin
$\therefore \arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2 = 0$

12. Ans: $6, \frac{2}{3}$

Sol: For equal roots,
$(2 + m)^2 - 4(m^2 - 4m + 4) = 0$
$4 + m^2 + 4m - 4m^2 + 16m - 16 = 0$
$3m^2 - 20m + 12 = 0$
$m = \frac{20 \pm \sqrt{400 - 144}}{6} = \frac{20 \pm 16}{6}$
$m = 6, \frac{2}{3}$

13. Ans: 5

Sol: The product of the roots is –16, which can be split as (–1, 16), (–16, 1), (2, –8), (–2, 8) and (4, –4)

No: of values = 5.

14. Ans: (- i)

Sol: Sum of roots = 1

Product of roots = (1 – i)

The other root is (- i)

15. Ans: (0, 1)

Sol: For roots with opposite signs, product must be negative

$\therefore \frac{a^2 - a}{3} < 0 \Rightarrow a^2 - a < 0$

a lies in (0, 1)

16. Ans: 4

Sol: For x < 0, $x^2 + 3x + 2 = 0 \Rightarrow$ two real roots

x > 0, $x^2 - 3x + 2 = 0 \Rightarrow$ two real roots

$\therefore$ Four real roots

17. Ans: $\frac{a-b}{a+b}$

Sol: $(x^2 - bx)(m + 1) = (m - 1)(ax - c)$

$(m + 1)x^2 - x[bm + b + am - a] + c(m - 1) = 0$

Sum of roots = 0

$\therefore m(a + b) + (b - a) = 0$

$m = \frac{a-b}{a+b}$.

18. Ans: 2 : 1

Sol: a + 8d = 0

$\frac{a+28d}{a+18d} = \frac{(a+8d)+20d}{(a+8d)+10d} = \frac{20}{10} = 2$

19. Ans: 10

Sol: $\frac{1}{S_1 S_2} = \left(\frac{1}{S_1} - \frac{1}{S_2}\right)\frac{1}{d}$ where d in the common difference

$\therefore$ The given expression

$= \left(\frac{1}{S_1} - \frac{1}{S_{101}}\right)\frac{1}{d} = \frac{1}{6}$

$\frac{S_{101} - S_1}{S_1 S_{101}} = \frac{d}{6} \longrightarrow (1)$

$S_1 + S_{101} = 50 \Rightarrow a + a + 100d = 50$

$a + 50d = 25 \longrightarrow (2)$

$(1) \Rightarrow \frac{100d}{a(S_{101})} = \frac{d}{6} \Rightarrow S_{101} = \frac{600}{a}$

Using (2) $a + S_{101} = 50$

$\therefore a + \frac{600}{a} = 50 \Rightarrow a = 30, 20$

For $a = 30 \Rightarrow d = \frac{1}{10}$

$a = 20 \Rightarrow d = \frac{1}{10}$

In either cares $\left|S_1 - S_{101}\right| = 100\left|\frac{1}{10}\right| = 10$

20. Ans: $n - 2 + \frac{1}{n-2}$

Sol: $a_1 = 0 \Rightarrow a_2 = d$ (Common difference)

$\therefore$ The given expression

$= \left(2 + \frac{3}{2} + \frac{4}{3} + ... + \frac{n-1}{n-2}\right) - \left(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n-3}\right)$

$= \underbrace{1 + 1 + ... + 1}_{n-3\,\text{times}} + \frac{n-1}{n-2}$

$= n - 3 + \frac{n-1}{n-2} = n - 3 + \frac{n-2+1}{n-2}$

$= n - 2 + \frac{1}{n-2}$

21. Ans: 66

Sol: $1^2 - 2^2 = 3(-1)$

$3^2 - 4^2 = 7(-1)$

$9^2 - 10^2 = 19(-1)$

$\therefore$ Required sum

$= -1(3 + 7 + 11 + 5 + 19) + 11^2$

$= -55 + 121 = 66$

22. Ans: 6

Sol: $S_{2n} = 3.S_n \Leftrightarrow \frac{\frac{2n}{2}(2a + (2n-1)d)}{\frac{n}{2}(2a + (n-1)d}$

$\Rightarrow 2a - (n + 1)d = 0. \Rightarrow 2a = (n + 1)d$

$k = \frac{S_{3n}}{S_n} = \frac{\frac{3n(2a + (3n-1)d}{2}}{\frac{n}{2}(2a + (n-1)d}$

$= \frac{3(4nd)}{(2nd)} = 6$

23. Ans: 8045

Sol: 9 – a = 2b = 3a – b – 9

$\therefore$ a + 2b = 9

3a – 3b = 9 $\Rightarrow$ a – b =3

$\therefore$ 3b = 6 $\Rightarrow$ b = 2 $\Rightarrow$ a = 5;

Common Difference = 4

$\therefore$ 2011th term = 5 + 2010 $\times$ 4

= 8045

24. Ans: 49

Sol: (n + 2) (n + 1) = 2550 = 51 $\times$ 50
$\Rightarrow$ n = 49

25. Ans: 3

Sol: $nC_{r-1} = 28$ $\quad nC_r = 56$ $\quad nC_{r+1} = 70$

$$\frac{n^1}{(r-1)^1(n-r+1)^1} = 28 \quad \frac{n^1}{r^1(n-r)^1} = 56$$

$$\frac{n^1}{(r+1)^1(n-r-1)^1} = 70$$

$$\frac{70}{56} = \frac{r^1(n-r)^1}{(r+1)(n-r-1)^1} = \frac{n-r}{r+1}$$

$$\frac{56}{28} = \frac{n-r+1}{r} = 2 \Rightarrow n = 3r - 1$$

$$\therefore \frac{5}{4} = \frac{3r-1-r}{r+1} = \frac{2r-1}{r+1}$$

5r+5 = 8r –4 $\Rightarrow$ 9 = 3r $\Rightarrow$ r = 3

26. Ans: 192

Sol: Number of numbers = 5! + numbers of 4 digit numbers beginning b, 7 or 8
= 5! + 3 $\times$ $^4P_3$
= 120 + 72
= 192

27. Ans: $\frac{3}{2}$

Sol: Put x = 1

$$\left(1-\frac{1}{3}\right)^{199} \times \left(1+\frac{1}{2}\right)^{200}$$

$$\left(\frac{2}{3}\right)^{199} \times \left(\frac{3}{2}\right)^{200} = \frac{3}{2}$$

28. Ans: $\frac{3}{2}$, 4

Sol: na = 6, $\frac{n(n-1)}{2}a^2 = \frac{27}{2} \Rightarrow (n-1)a = \frac{9}{2}$

$\therefore$ 6 – a = $\frac{9}{2} \Rightarrow a = \frac{3}{2}$, n = 4

29. Ans: $\frac{1}{(n+1)(n+2)}$

Sol: $x(1-x)^n = C_0x - C_1x^2 + C_2x^3 \,.+(-1)^4\, C_n x^{n+1}$

$$\int_0^1 x(1-x)^n dx = \left[\frac{C_0x^2}{2} - \frac{C_1x^3}{3} + \frac{C_2x^4}{4} + . + (-1)c_n\frac{x^{n+2}}{n+2}\right]_0^1$$

$$\therefore \frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - \frac{C_3}{5} + ... + (-1)^n\frac{C_n}{n+2}$$

$$= \int_0^1 x(1-x)^n dx$$

$$= \int_0^1 \left(x^n - x^{n+1}\right)dx$$

$$= \frac{1}{n+1} - \frac{1}{n+2} = \frac{1}{(n+1)(n+2)}$$

30. Ans: -2, -1

Sol: $$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

a + 4 +2b = 0
2a + 2 –2b = 0
$\Rightarrow$ a = –2 and b = –1

31. Ans: -3

Sol: $$A^{-1} = \frac{1}{2x}\begin{bmatrix} x & -1 \\ 0 & 2 \end{bmatrix}$$

$\therefore$ comparing $\frac{-1}{2x} = \frac{1}{6}$

$\therefore$ x = -3

32. Ans: $A^{10} = \begin{bmatrix} \cos 10\alpha & \sin 10\alpha \\ -\sin 10\alpha & \cos 10\alpha \end{bmatrix}$

Sol: By induction $A^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$

$\therefore$ $A^{10} = \begin{bmatrix} \cos 10\alpha & \sin 10\alpha \\ -\sin 10\alpha & \cos 10\alpha \end{bmatrix}$

33. Ans: 7 A^8

Sol: $A^2 = I$
$\therefore$ $A^2 + 2A^4 + 4A^6 = 7I = 7A^8$

34. Ans: –2

Sol: $$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix} = I$$

$\Rightarrow$ x = 0
$\therefore$ $x^2 + x - 2 = -2$

35. Ans: $\frac{c-b}{a+b+c}$

Sol: Adding 2(a + b + c) (x + y + z)
– (a + b + c) (x + y + z) = 0
ie., x + y + z = 0 ($\because$ a + b + c $\neq$ 0)
$\therefore$ for (1)
(b + c) (- x) – a x = b – c

$$\therefore x = -\frac{b-c}{a+b+c} = \frac{c-b}{a+b+c}$$

36. Ans: $\left(-\frac{2}{3}, 8\right)$

Sol: $4x^2 - 12x + 9 < x^2 + 10x + 25$
$3x^2 - 22x - 16 < 0$
$3x^2 - 24x + 2x - 16 < 0$

$3x(x-8)+2(x+8)<0$
$(3x+2)(x-8)<0$
$\Rightarrow\Rightarrow x\in\left(-\frac{2}{3},8\right)$

37. Ans: $(-\infty, 2)\cup[7,\infty)$.

Sol: $\frac{x+3}{x-2}-2\le 0$
$\frac{7-x}{x-2}\le 0$
$\therefore\ x\ne 2$ and $(x-2)(x-7)\ge 0$
$\therefore\ x\in(-\infty,2)\cup[7,\infty)$.

38. Ans: Roses are not red or the sun is a star

Sol: Obvious

39. Ans: $\sim q\vee\sim p$

Sol: $p\to\sim q\ =\sim p\vee\sim q$
$=\sim q\vee\sim p$

40. Ans: $(\sim p\wedge q)\vee\sim q$

Sol: $\sim[(p\vee\sim q)\wedge q]\ =\sim(p\vee\sim q)\vee\sim q$
$=(\sim p\wedge q)\vee\sim q$

41. Ans: 0

Sol: $\text{Cos }20^0-\text{Cos }80^0-\text{Cos }40^0$
$=\text{Cos }20^0-2\,\frac{1}{2}.\ \text{Cos }20^0\ =0$

42. Ans: 1

Sol: $\dfrac{\cos^2\frac{\pi}{4}-\sin^2\theta}{\sin^2\frac{\pi}{4}-\sin^2\theta}=1$

43. Ans: 0

Sol: $\sin(\theta+\alpha)\cos\alpha-\cos(\theta+\alpha)\sin\alpha$
$=3[\sin(\theta+\alpha)\cos\alpha+\cos(\theta+\alpha)\sin\alpha)$
$2\sin(\theta+\alpha)\cos\alpha=-4\cos(\theta+\alpha)\sin\alpha$
$\therefore\ \tan(\theta+\alpha)+2\tan\alpha=0$

44. Ans: $\cot\beta$

Sol: $\dfrac{2\cos\frac{\alpha+\gamma}{2}.\sin\frac{\alpha-\gamma}{2}}{-2\sin\frac{\alpha+\gamma}{2}.\sin\frac{\gamma-\alpha}{2}}$
$=\cot\beta$ since $\frac{\alpha+\gamma}{2}=\beta$.

45. Ans: $\frac{\sqrt{3}}{2}$

Sol: $3\sin^{-1}x=\pi$
$\Rightarrow\ x=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

46. Ans: $\sin^4\theta$.

Sol: $\frac{1}{8}(2-4\cos 2\theta+2\cos^2 2\theta)$
$\frac{1}{4}(1-2\cos 2\theta+\cos^2 2\theta)$
$\frac{1}{4}(1-\cos 2\theta)^2=\frac{1}{4}\times 4\sin^4\theta=\sin^4\theta$.

47. Ans: $-\frac{31}{8}$

Sol: $8\cos^2 2\theta-65\cos 2\theta+8=0$
$8\cos^2 2\theta-64\cos 2\theta-\cos 2\theta+8=0$
$8\cos 2\theta(\cos 2\theta-8)-(\cos 2\theta-8)=0$
$(8\cos 2\theta-1)(\cos 2\theta-8)=0$
$\therefore\ \cos 2\theta=\frac{1}{8}$
$4\cos 4\theta=-4(1-2\cos^2 2\theta)$
$=-4\left(1-\frac{2}{64}\right)$
$=-\frac{31}{8}$

48. Ans: $\frac{3\pi}{4}$

Sol: it is $\pi+\tan^{-1}\frac{2+3}{1-6}$
$=\frac{3\pi}{4}$

49. Ans: $2\le k\le 6$

Sol: $k\sin x+1-2\sin^2 x=2k-7$
$2\sin^2x-k\sin x+2k-8=0$
$\sin x=2$ is an in admissible root
The other root is $\frac{k-4}{2}$
$\therefore\ |k-4|\le 2$
$-2\le k-4\le 2$
$2\le k\le 6$

50. Ans: $2\left|a\sin\frac{\alpha-\beta}{2}\right|$

Sol:
$|a|.\sqrt{4\sin^2\frac{\alpha+\beta}{2}.\sin^2\frac{\alpha-\beta}{2}+4\cos^2\frac{\alpha+\beta}{2}.\sin^2\frac{\alpha-\beta}{2}}$
$=2\left|a\sin\frac{\alpha-\beta}{2}\right|$

51. Ans: 5

Sol: AC = BD = $\sqrt{3^2+4^2}$ = 5

52. Ans: – 2

Sol: y – 2 = $\dfrac{2-1}{1+\frac{1}{2}}$ = (x – 1)

= $\dfrac{2}{3}$ (x – 1)

∴ x intercept = – 3 + 1 = – 2

53. Ans: below the x – axis at distance of $\dfrac{3}{2}$

Sol: ax + 2by + 3b +λ (b × – 2ay – 3a) = 0

When λb + a = 0 ⇒ λ = $-a/b$

Y – intercept = $-\dfrac{3b-3\lambda a}{2b-2\lambda a}$

$= -\dfrac{3(b^2+a^2)}{2(b^2-c^2)} = \dfrac{-3}{2}$

54. Ans: $\dfrac{1}{p^2}+\dfrac{1}{q^2}$

Sol: obviously

55. Ans: x – y = 3

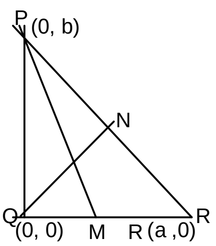
Sol: slope :1, pt (2, – 1)

line is y + 1 = x – 2

⇒ x – y –3 = 0

56. Ans: $a^2 = 2b^2$

Sol:



Slope pf PM = $\dfrac{-2b}{a}$

Slope of QN = $\dfrac{b}{a}$

∴ $-\dfrac{-2b}{a}.\dfrac{b}{a} = -1$

⇒ $2b^2 = a^2$

57. Ans: 0

Sol: Slope of the line parallel to x – axis

∴ slope = 0

58. Ans: $\sqrt{\mu-\lambda}$

Sol: Length of athe tangent from (x, y) to second circle = $\sqrt{x_1^2+y_1^2+2fy_1+\mu}$

But $x_1^2+y_1^2+2fy_1 = -\lambda$

∴ = $\sqrt{\mu-\lambda}$

59. Ans: 20

Sol: Point (4, – 3) lies outside the circle

∴ Radius of circle = 6

Distance from centre (– 2, 5) to (4, –3)

Is 10

∴ Maximum distance = 10 + 6 = 16

Minimum distance = 10 – 6 = 4

∴ sum = 16 + 4 = 20

60. Question wrong.

However if the given equation is "$x^2 + y^2 - 6x + 2y = 0$", the answer would be x + 3y = 0.

61. Ans: $x = -\dfrac{1}{2}+\cos\theta \quad y = \dfrac{-\sqrt{3}}{2}+\sin\theta$.

Sol: Given equation is

$$\left(x+\frac{1}{2}\right)^2+\left(y+\frac{\sqrt{3}}{2}\right)^2 = 1$$

∴ parametric equation is

$x = -\dfrac{1}{2}+\cos\theta \quad y = \dfrac{-\sqrt{3}}{2}+\sin\theta$.

62. Ans: $8\sqrt{3}$

Sol: $y = \dfrac{1}{\sqrt{3}}x$, $y^2 = 4x$

$\dfrac{x^2}{3} = 4x$

∴ x = 12 (x ≠0)

∴ point is $\left(12, \pm 4\sqrt{3}\right)$

∴ side = $\sqrt{144+48}$

= $\sqrt{192}$

= $8\sqrt{3}$.

63. Ans: 8x – 5 = 0.

Sol: $y^2 = 4\left(\dfrac{5}{8}\right)x$

Equation is x = $\dfrac{5}{8}$ ie 8x – 5 = 0.

64. Ans: 2

Sol: clearly the point $(\pm 2, 0)$ are the foci

"2a" = 8, "2ae" = 4

$\therefore a^2 - b^2 = 4$

$a^2 = 16$

$b^2 = 12$

ellipse is $\frac{x^2}{16} + \frac{y^2}{12} = 1$

$\frac{x^2}{16} + \frac{9}{12} = 1$

$\Rightarrow x^2 = 4 \Rightarrow x = \pm 2$.

65. Ans: $\left(-\frac{7}{2}, 1\right)$

Sol: $y^2 + 6x - 2y + 13 = 0$

$(y-1)^2 = -6x - 12$

$= 6(-x-2)$

$Y^2 = -4\left(\frac{3}{2}\right)(X)$

focus is $\left(\frac{-3}{2}, 0\right)$

ie $\left(-\frac{7}{2}, 1\right)$.

66. Ans: $2\sqrt{5}$

Sol: $y = x^2 - 4x + 3$

$y + 1 = (x-2)^2$

vertex is (2, –1)

$x^2 = 9 - (y-3)^2$

centre is (0, 3)

distance = $\sqrt{4+16}$

$= 2\sqrt{5}$.

67. Ans: $|a|^4 b$.

Sol: $\bar{a} \times \{a \times \{a \times (a \times b)\}\}$

$= a \times \left\{a \times \left((a.b)a - |a^2|b\right)\right\}$

$= a \times \left\{-|a|^2 a \times b\right\}$ (a.b) = 0

$= -|a|^2 a \times (a \times b)$

$= -|a|^2\left((a.b)a - |a|^2 b\right)$

$= |a|^4 b$.

68. Ans: –6

Sol: $\sqrt{64 + a^2} = 10$

$a^2 = 100 - 64 = 36$

$\therefore a = \pm 6$

But $8i + aj$ has same direction $-4i - 3j$

$\therefore a = -6$.

69. Ans: –5

Sol: $\bar{a}.m\bar{b} = 120$

$\Rightarrow m(-24) = 120 \Rightarrow m = -5$.

70. Ans: 90°

Sol: clearly $\bar{a}, \bar{b}, \bar{c}$ from triangle

$\therefore$ angle between

$\bar{a}$ and $\bar{b} = 180 - (65 + 25) = 90°$.

71. Ans: $2\hat{i} + 3\hat{j} + \hat{k}$

Sol: Vertex A = $2i + 6j + 4k$

Centroid = $2i + 4j + 2k$

D divides AG externally in the ratio 3 : 1

$\therefore$ D is $\frac{3(2i+4j+2k) - (2i+6j+4k)}{3-1}$

$= 2i + 3j + k$.

72. Ans: 3

Sol: $\frac{2 - 2a - 1}{\sqrt{6}} = \frac{-5}{\sqrt{6}}$

$\Rightarrow a = 3$.

73. Ans: $\frac{1}{4}\left[\left(\sqrt{6}+\sqrt{2}\right)\hat{i} + \left(\sqrt{6}-\sqrt{2}\right)\hat{j}\right]$

Sol: $\frac{x+y}{\sqrt{2}} = \frac{\sqrt{3}}{2}$

$\frac{x-y}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$x + y = \frac{\sqrt{6}}{2}$

$x - y = \frac{\sqrt{2}}{2}$

$x = \frac{\sqrt{6}+\sqrt{2}}{4}$

$y = \frac{\sqrt{6}-\sqrt{2}}{4}$.

74. Ans: 0°

Sol: $(2i + 3j + 4k)(i + 2j - 2k) = 2 + 6 - 8 = 0$

$\therefore$ line is perpendicular to the normal to Plane

$\therefore$ line is parallel to the plane

Angle is 0.

75. Ans: (4, 0, –1)

Sol: $1 + 3\lambda = 4 + 2\mu$

$1 - \lambda = 0 \Rightarrow \lambda = 1.\ \mu = 0$

And the point is (4, 0, –1)

76. Ans: 6x + 3y + 2z – 6 = 0

Sol: Obviously

77. Ans: 17

Sol: Length = $\sqrt{9^2+12^2+8^2}$

$= \sqrt{15^2+8^2} = 17.$

78. Ans: $\frac{x-1}{1} = \frac{y}{-2} = \frac{z+2}{5}$

Sol: Obviously

79. Ans: $3x - 2y + 4z = 11$

Sol: $3(x-1) - 2(y-2) + 4(z-3) = 0$

$3x - 2y + 4z - 11 = 0.$

80. Ans: 0

Sol: $\begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & 0 \\ k & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow k = 0.$

81. Ans: perpendicular to x-axis

Sol: Direction ratios of the line : (0, 1, 2)

Direction ratios of X axis : (1, 0, 0)

$\therefore$ perpendicular to X axis.

82. Ans: 25

Sol: Total = $9 \times 15 = 135$

$135 + x = 160$

$x = 160 - 135$

$= 25.$

83. Ans: $\frac{50}{101}$

Sol: $\frac{\frac{99\times100}{2}+x}{100} = 100x$

$99 \times 50 + x = 100x \times 100$

$9999x = 99 \times 50$

$x = \frac{99\times50}{9999} = \frac{50}{101}$.

84. Ans: 24

Sol: $\frac{x}{5}, \frac{x}{4}, \frac{x}{3}, \frac{x}{2}, x$

$\frac{x}{3} = 8$

$x = 24.$

85. Ans: 27.1

Sol: 10×2.71

86. Ans: $[-3, -2]$

Sol: $-1 \le \log_2\left(x^2+5x+8\right) \le 1$

$\Rightarrow 2^{-1} \le x^2 + 5x + 8 \le 2$

<u>Case 1 :</u>

$x^2 + 5x + 8 \ge \frac{1}{2}$

$\Rightarrow \left(x+\frac{5}{2}\right)^2 + \frac{7}{4} \ge \frac{1}{2}$

It is true $\forall\ x \in R$.

<u>Case 2</u>

$x^2 + 5x + 8 \le 2$

$\Rightarrow x^2 + 5x + 6 \le 0$

$\Rightarrow (x + 2)(x + 3) \le 0$

$\Rightarrow x \in [-3, -2]$

87. Ans: 20

Sol: $\underset{x\to0}{\text{Lt}} \frac{(1+2x)^{10}-1}{x} = 20(1 + 2x)^9 = 20.$

88. Ans: $[2\log_e 2, \infty)$

Sol: $f(x) = \log_e\left(3x^2+4\right)$

$y = \log_e\left(3x^2+4\right)$

$3x^2 + 4 = e^y$

$3x^2 = e^y - 4$

$x^2 = \frac{e^y-4}{3}$

$x = \sqrt{\frac{e^y-4}{3}}$

$e^y - 4 \ge 0$

$e^y \ge 4$

$y \ge \log_e 4 = 2\log_e 2.$

89. Ans: $\frac{100}{77} \times 2^{23}$

Sol: $\underset{x\to2}{\text{Lt}} \frac{x^{100}-2^{100}}{x^{77}-2^{77}} = \frac{100x^{99}}{77x^{76}}$

$= \frac{100\times2^{99}}{77\times2^{76}}$

$= \frac{100}{77}\times2^{99-76}$

$= \frac{100}{77}\times2^{23}$.

90. Ans: $\frac{1}{4}$

Sol: $\lim_{k\to\infty} \frac{K^2(k+1)^2}{4K^4} = \lim_{k\to\infty} \frac{k^4\left(1+\frac{1}{k}\right)^2}{4k^4}$

$= \frac{1}{4}.$

91. Ans: $-\frac{1}{2}$

Sol: $y = \frac{1}{\operatorname{cosec}^2 \cot^{-1}\sqrt{\frac{1+x}{1-x}}}$

$= \frac{1}{1+\cot^2 \cot^{-1}\sqrt{\frac{1+x}{1-x}}} = \frac{1}{1+\frac{1+x}{1-x}}$

$= \frac{1-x}{1-x+1+x} = \frac{1-x}{2}$

$\frac{dy}{dx} = \frac{-1}{2}.$

92. Ans: $\frac{1}{3}$

Sol: $x = 3\sin^{-1}t$

$\frac{dx}{dt} = \frac{3\times 1}{\sqrt{1-t^2}}$

$y = \sin^{-1}t \qquad \frac{dy}{dt} = \frac{1}{\sqrt{1-t^2}}$

$\frac{dy}{dx} = \frac{1}{3}.$

93. Ans: 274

Sol: At x = 0

$\frac{dy}{dx} = (x + 1)(x + 2)(x + 3)(x + 4) \to 24$

$+ (x + 1)(x + 2)(x + 3)(x + 5) \to 30$

$+ (x + 1)(x + 2)(x + 4)(x + 5) \to 40$

$+ (x + 1)(x + 3)(x + 4)(x + 5) \to 60$

$+(x + 2)(x + 3)(x + 4)(x + 5) \to 120$

= 274

94. Ans: $-\frac{1}{2}$

Sol: $y = \cot^{-1}\tan\left(\frac{x}{2}\right) = \cot^{-1}\cot\left(\frac{\pi}{2}-\frac{x}{2}\right)$

$= \frac{\pi}{2}-\frac{x}{2}$

$\frac{dy}{dx} = \frac{-1}{2}.$

95. Ans: 2

Sol: $y = \left(\sin^{-1}x\right)^2$

$\frac{dy}{dx} = 2\sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}\,\frac{dy}{dx} = 2\sin^{-1}x$

$\sqrt{1-x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\times\frac{-2x}{2\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$

$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2.$

96. Ans: -1

Sol: $x^y y^x = 16$

ylogx + xlogy = log16

$\frac{y}{x} + \log x\frac{dy}{dx} + \log y + \frac{x}{y}\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left(\log x + \frac{x}{y}\right) = \frac{y}{x} - \log y$

at (2, 2) $\frac{dy}{dx}(\log 2 + 1) = -1 - \log 2$

$\frac{dy}{dx} = -1.$

97. Ans: $2\cos(2y) - 5$

Sol: sin2y = x + 5y

$\Rightarrow x = \sin 2y - 5y$

$\Rightarrow \frac{dx}{dy} = 2\cos 2y - 5.$

98. Ans: 416

Sol: $R(x) = 13x^2 + 26x + 15$

$\frac{d}{dx}R(x) = 26x + 26$

Marginal revenue at x = 15

$= 26 \times 15 + 26$

$= 390 + 26 = 416.$

99. Ans: $(0, 1) \cup (2, \infty)$

Sol: $f(x) = x^2(x-2)^2$

$f'(x) = x^2\, 2(x-2) + (x-2)^2 \times 2x$

$= 2x(x-2)(x + x - 2)$

$= 2x(x-2)(2x-2)$

$= 4x(x-2)(x-1)$

> 0

$(0, 1) \cup (2, \infty)$

100. Ans: (6,7)

Sol: $\frac{dy}{dx} = \frac{4}{2\sqrt{4x+1}}$

$= \frac{2}{\sqrt{4x+1}}$

$\frac{2}{\sqrt{4x+1}} = \frac{2}{5}$

$\Rightarrow 4x + 1 = 25$

$\Rightarrow 4x = 24$

$\Rightarrow x = 6$

$\Rightarrow y = 2 + \sqrt{24+1}$

$= 2+5$

$= 7$

$\therefore$ The required point is (6,7)

101. Ans: $y = \frac{1}{4}$

Sol: $y = \frac{1}{(x^2+2x+5)^2}$

$\frac{dy}{dx} = -\frac{1(2x+2)}{\left(x^2+2x+5\right)^2}$

$= 0 \Rightarrow x = -1$

$y = \frac{1}{(-1)^2 + 2(-1) + 5}$

$= \frac{1}{4}$

102. Ans: $9x + y - 6 = 0$

Sol: $x = \frac{t-1}{t+1}$ $y = \frac{t+1}{t-1}$

$\Rightarrow \frac{dx}{dt} = \frac{(t+1)-(t-1)}{(t+1)^2} = \frac{2}{(t-1)^2}$

$\& \frac{dy}{dt} = \frac{(t-1)(t+1)}{(t+1)^2} = \frac{12}{(t-1)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{(t-1)^2} \frac{(t+1)^2}{2}$

$= -\left[\frac{t+1}{t-1}\right]^2$

$\Rightarrow \left.\frac{dy}{dx}\right/_{t=2} = -\left[\frac{3}{1}\right]^2$

$= -9$

$x = \frac{1}{3}$ & $y = 3$ at $t = 2$

$\therefore$ Equation of tangent at t=2 is given by,

$y - 3 = -9\left(x - \frac{1}{3}\right) = -3(3x-1)$ or

$9x+y-6 = 0$

103. Ans: (–6, 3)

Sol: All the points in the given options are points of the conic.
Distance of the point (–6, 3) to the line
$3x + 2y + 1 = 0$ is minimum.

104. Ans: $\frac{25}{4}$

Sol: $\frac{1}{2\sqrt{x}} = \frac{3-2}{5}$

$\rightarrow x = \frac{25}{4}$

105. Ans: $2\tan^{-1}\sqrt{x} + C$

Sol: $\sqrt{x} = t$

$\Rightarrow 2\int \frac{dt}{t^2+1}$

$= 2\tan^{-1}\sqrt{x} + C$

106. Ans: $-\frac{\log x}{x} - \frac{1}{x} + C$

Sol: $\cos x = t$

$\Rightarrow \int te^{-t}dt$

$= \frac{te^{-t}}{-1} - \int \frac{e^{-t}}{-1}dt$

$= -te^{-t} - e^{-t} + C$

$= -\frac{\log x}{x} - \frac{1}{x} + C$.

107. Ans: tanx

Sol: $\frac{f(x)}{\log\cos x} = \frac{-1}{\log\cos x} \times \frac{-\sin x}{\cos x}$

$= \frac{\tan x}{\log\cos x}$

$\Rightarrow f(x) = \tan x$

108. Ans: $x - \sqrt{1-x^2}\,\sin^{-1}x + C$

Sol: $\sin^{-1}x = t \Rightarrow$

$I = \int t\sin t\,dt$

$= -t\cos t - \int -\cos t\,dt$

$= -t\cos t + \sin t + C$

$= -\sqrt{1-x^2}\,\sin^{-1}x + x + C$

109. Ans: $-\frac{3}{2}x + \frac{35}{36}\log\left|9e^{2x} - 4\right| + C$

Sol: $\frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} = \frac{4e^{2x}+6}{9e^{2x}-4}$

$4e^{2x} + 6 = A(18e^{2x}) + B(9e^{2x} - 4)$

$\Rightarrow -4B = 6$ or $B = -\frac{3}{2}$

and $18A + 9B = 4$

$\Rightarrow 18A - \frac{27}{2} = 4$

$\Rightarrow A = \frac{1}{18}\left[4 + \frac{27}{2}\right] = \frac{35}{36}$

$\therefore \int \frac{(4e^x + 6e^{-x})dx}{9e^x - 4e^{-x}} = \frac{35}{36}\int \frac{18e^{2x}}{9e^{2x}-4}dx +$

$$\frac{-3}{2}\int dx$$

$$= \frac{35}{36}\log\left|9e^{2x}-4\right| - \frac{3}{2}x + C .$$

110. Ans: $\sin^{-1}x + \sqrt{1-x^2} + C$

Sol: $\int \frac{(1-x)dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} + \frac{1}{2}\int \frac{-2xdx}{\sqrt{1-x^2}}$

$= \sin^{-1}x + \sqrt{1-x^2} + C$.

111. Ans: $\frac{x}{2} + \frac{1}{2}\log|\cos x + \sin x| + C$

Sol: $\int \frac{\cos x dx}{\cos x + \sin x} dx$

$= \frac{1}{2}\left[\int dx + \int \frac{\cos x - \sin x}{\cos x + \sin x}\right]$

$= \frac{x}{2} + \frac{1}{2}\log|\cos x + \sin x| + C$

112. Ans: -1

Sol: $-\int_a^0 (x+1)dx = -\frac{1}{2}$

$\left(\frac{x^2}{2} + x\right)_a^0 = \frac{1}{2}$

$\Rightarrow -\frac{a^2}{2} - a = \frac{1}{2}$

$\Rightarrow a^2 + 2a + 1 = 0$

$\Rightarrow (a+1)^2 = 0$

$\Rightarrow a = -1$

113. Ans: $\frac{1}{42}$

Sol: $\int_0^1 x(1-x)^5 dx = \int_0^1 (1-x)x^5 dx$

$= \int_0^1 \left(x^5 - x^6\right)dx = \left[\frac{x^6}{6} - \frac{x^7}{7}\right]_0^1$

$= \frac{1}{6} - \frac{1}{7} = \frac{1}{42}$.

114. Ans: 3

Sol: $\int_0^2 |x-2|dx = \int_0^2 (2-x)dx = \left[2x - \frac{x^2}{2}\right]_0^2$

$= 4 - \frac{4}{2} = 2$

$\int_0^2 [x]dx = \int_0^1 [x]dx + \int_1^2 [x]dx = 0 + \int_1^2 1dx = 1$

$\therefore \int_0^2 (|x-2| + [x])dx = 2 + 1 = 3$.

115. Ans: $\frac{1}{25} - \frac{6e^{-5}}{25}$

Sol: $\int_0^1 xe^{-5x}dx = \left(\frac{xe^{-5x}}{-5}\right)_0^1 - \int_0^1 \frac{e^{-5x}}{-5}dx$

$= -\frac{1}{5}e^{-5} + \frac{1}{5}\left(\frac{e^{-5}}{-5}\right)_0^1$

$= -\frac{e^{-5}}{5} - \frac{1}{25}\left[e^{-5} - 1\right]$

$= \frac{1}{25} - e^{-5}\left[\frac{1}{5} + \frac{1}{25}\right]$

$= \frac{1}{25} - e^{-5}\left[\frac{5+1}{25}\right]$

$= \frac{1}{25} - \frac{6e^{-5}}{25}$.

116. Ans: 4

Sol: Area $= 4\int_0^{\pi/2} \sin x dx = 4[-\cos x]_0^{\pi/2}$

$= 4.$

117. Ans: order 1; degree 3

Sol: $y^2 = 2c\left(x + \sqrt{c}\right)$ ----- (1)

$\Rightarrow y^2 = 2cx + 2c\sqrt{c}$

$\Rightarrow 2y\frac{dy}{dx} = 2c \Rightarrow c = y\frac{dy}{dx} = yy_1$

Substituting in (1)

$y^2 = 2yy_1\left[x + \sqrt{yy_1}\right]$

$\Rightarrow y^2 = 2xyy_1 + 2(yy_1)^{3/2}$

$\Rightarrow \left[y^2 - 2xyy_1\right]^2 = 4(yy_1)^3$

$\therefore$ order = 1, degree = 3.

118. Ans: $\sqrt{1+x^2}$

Sol: $\left(1+x^2\right)\frac{dy}{dx} + xy = x$

$\Rightarrow \frac{dy}{dx} + \left(\frac{x}{1+x^2}\right)y = \frac{x}{\left(1+x^2\right)}$

I.F $= e^{\int \frac{x}{1+x^2}} = e^{\frac{1}{2}\log\left(1+x^2\right)} = \sqrt{1+x^2}$.

119. Ans: $x^2y + 1 = 3x$

Sol: $x\frac{dy}{dx} + y = \frac{1}{x^2}$

$\Rightarrow \frac{d}{dx}(xy) = \frac{1}{x^2}$

$\Rightarrow xy = \int \frac{1}{x^2}dx = -\frac{1}{x} + C$

Substituting x = 1, y = 2

$\Rightarrow 2 = \frac{-1}{1} + C \quad \Rightarrow C = 3$

$\therefore xy = \frac{-1}{x} + 3$

$\Rightarrow x^2y + 1 = 3x$

120. Ans: $e^{-y} = e^{-x} - e^{x} - x^2 + C$

Sol: $\frac{dy}{dx} = e^{y}\left[e^{x} + e^{-x} + 2x\right]$

$\int e^{-y}dy = \int \left(e^{x} + e^{-x} + 2x\right)$

$\Rightarrow -e^{-y} = e^{x} - e^{-x} + x^2 + C$

$\Rightarrow e^{-y} = e^{-x} - e^{x} - x^2 + C.$