MATHEMATICAL SCIENCES

Subject code – 4 **Booklet Code – C**

2011 (I)

MATHEMATICAL SCIENCES
TEST BOOKLET

(19 June 2011)

Time: 3:00 Hours **Maximum Marks: 200**

INSTRUCTIONS

1. You have opted for English as medium of Question Paper. This Test Booklet contains one hundred and twenty (20 Part 'A' + 40 Part 'B' + 60 Part 'C') Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 25 and 20 questions from part 'A', 'B' and 'C' respectively. If more than required number of questions are answered, only first 15, 25 and 20 questions in Part 'A', 'B' and 'C' respectively, will be taken up for evaluation.
2. Answer sheet has been provided separately. Before you start filling up your particulars, please ensure that the booklet contains requisite number of pages and that these are not torn or mutilated. If it is so, you may request the Invigilator to change the booklet. Likewise, check the answer sheet also. Sheets for rough work have been appended to the test booklet.
3. Write your Roll No., Name, Your address and Serial Number and this Test Booklet on the Answer sheet in the space provided on the side 1 of Answer sheet. Also put your signatures in the space identified.
4. You must darken the appropriate circles related to Roll Number, Subject Code, Booklet Code and Centre Code on the OMR answer sheet. It is the sole responsibility of the candidate to meticulously follow the instructions given on the Answer Sheet, failing which, the computer shall not be able to decipher the correct details which may ultimately result in loss, including rejection of the OMR answer sheet.
5. Each question in Part 'A' carries 2 marks, Part 'B', 3 marks and Part 'C' 4.75 marks respectively. There will be negative marking @0.5 marks in Part 'A' and @0.75 in Part 'B' for each wrong answer and no negative marking for part 'C.
6. Below each question in Part 'A' and 'B', four alternatives or responses are given. Only one of these alternatives is the "correct" option to the question. You have to find, for each question, the correct or the best answer. In Part 'C' each question may have 'ONE' or 'MORE' correct options. Credit in a question shall be given only on identification of 'ALL' the correct options in Part 'C'. No credit shall be allowed in a question if any incorrect option is marked as correct answer.
7. Candidates found copying or resorting to any unfair means are liable to be disqualified from this and future examinations.
8. Candidate should not write anything anywhere except on answer sheet or sheets for rough work.
9. After the test is over, you MUST hand over the answer sheet (OMR) to the invigilator.
10. Use of calculator is not permitted.

Roll No. ....

Name ....

I have verified all the information filled in by the candidate.

....

Signature of the Invigilator

PART-A

(1.) A physiological disorder X always leads to the disorder Y. However, disorder Y may occur by itself. A population shows 4% incident of disorder Y. Which of the following inferences in valid?

1. 4% of the population suffers from both X & Y
2. Less than 4% of the population suffers from X
3. At least 4% of the population suffers from X
4. There is no incident of X in the given population.

(2.) Exposing an organism to a certain chemical can change nucleotide bases in a gene, causing mutation. In one such mutated organism if a protein had only 70% of the primary amino acid sequence, which of the following is likely?

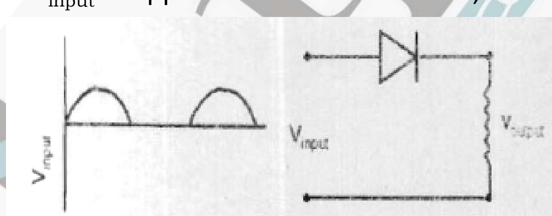
1. Mutation broke the protein
2. The organism could not make amino acids
3. Mutation created a terminator codon
4. The gene was not transcribed

(3.) The speed of a car increases every minute as shown in the following Table. The speed at the end of the 19th minute would be

Time (minutes)	Speed (m/sec)
1	1.5
2	3.0
3	4.5
24	36.0
25	37.5

1. 26.5
2. 28.0
3. 27.0
4. 28.5

(4.) If V_{input} is applied to the circuit shown, the output would be



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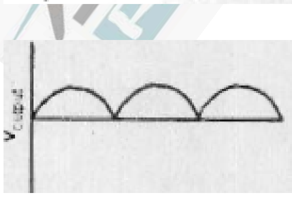

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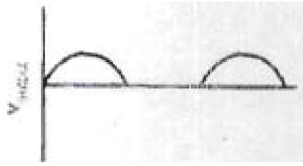

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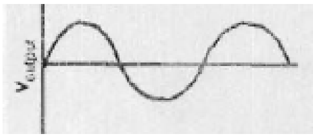
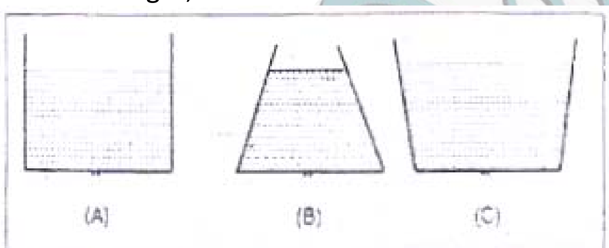


(5.) Water is dripping out of a tiny hole at the bottom of three flask whose base diameter is the same, and are initially filled to the same height, as shown



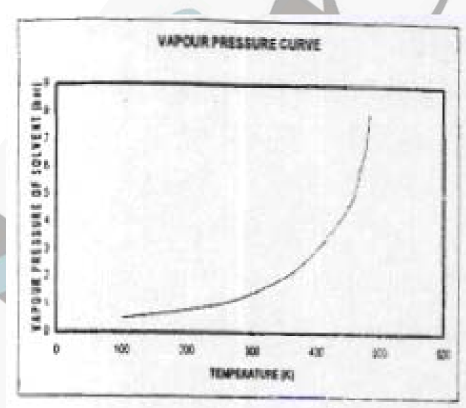
Which is the correct comparison of the rate of fail of the volume of water in the three flask?

1. A fastest, B slowest
2. B fastest, A slowest
3. B fastest, C slowest
4. C fastest, B slowest.

(6.) A reference material is required to be prepared with 4 ppm calcium. The amount of $CaCO_3$ (molecular weight = 100) required to prepare 1000 g of such a reference material is

1. 10 μg
2. 4 μg
3. 4 mg
4. 10 mg

(7.)



The normal boiling point of a solvent (whose vapour pressure curve is shown in the figure) on a planet whose normal atmospheric pressure is 3 bar, is about

1. 400 K
2. 273 K
3. 100 K
4. 500 K.

(8.) How many σ bonds are present in the following molecule?

$HC \equiv CCH = CHC_3$

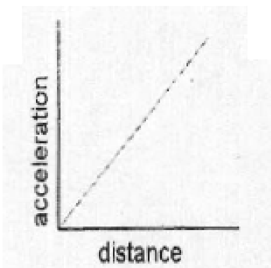
1. 6

2. 10
3. 4
4. 13

(9.) The reason for the hardness of diamond is
1. Extended covalent bonding
2. Layered structure
3. Formation of cage structures
4. Formation of tubular structures.

(10.) The acidity of normal rain water is due to
1. SO_2
2. CO_2
3. NO_2
4. NO

(11.) A ball is dropped from a height h above the surface of the earth. Ignoring air drag, the curve that best represents its variation of acceleration is

1.


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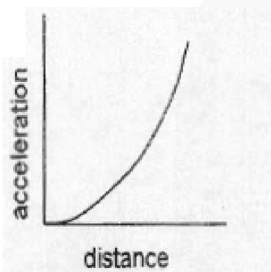

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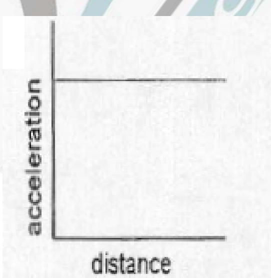
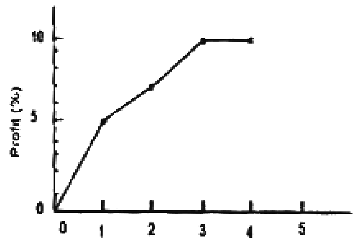


(12.)



The cumulative profits of a company since its inception are shown in the diagram. If the net worth of the company at the end of 4th year is 99 crores, the principal it had started with was

1. 9.9 crores
2. 91 crores
3. 90 crores
4. 9.0 crores

(13.) Diabetic patients are advised a low glyacemic index diet. The reason for this is

1. They require less carbohydrate then healthy individuals
2. They cannot assimilate ordinary carbohydrates
3. They need to have slow, but sustained release of glucose in their blood stream
4. They can tolerate lower, but not higher than normal blood sugar levels.

(14.) Standing on a polished stone floor one feels colder than on a rough floor of the same stone. This is because

1. Thermal conductivity of the stone depends on the surface smoothness
2. Specific heat of the stone changes by polishing it
3. The temperature of the polished floor is lower than that of the rough floor
4. There is greater heat loss from the soles of the feet when in contact with the polished floor than with the rough floor.

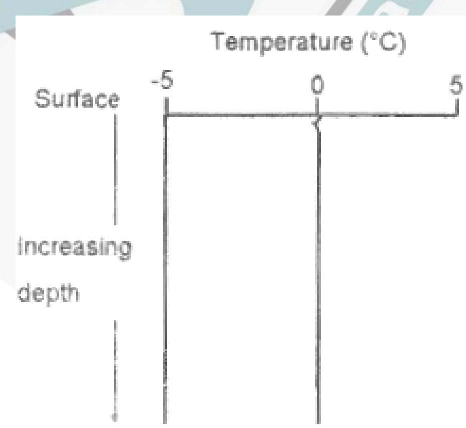
(15.) Popular use of which of the following fertilizers increases the acidity of soil?

1. Potassium Nitrate
2. Ammonium sulphate
3. Urea
4. Superphosphate of lime

(16.) If the atmospheric concentration of carbon dioxide is doubled and there are favourable conditions of water, nutrients, light and temperature, what would happen to water requirement of plants?

1. It decreases initially for a short time and then returns to the original value
2. It increases
3. It decreases
4. It increases initially for a short time and then returns to the original value.

(17.)



The graph represents the depth profile of temperature in the open ocean; in which region this is likely to be prevalent?

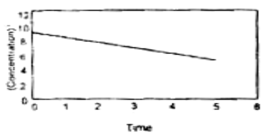
1. Tropical region
2. Equatorial region
3. Polar region
4. Sub-tropical region.

(18.) Glucose molecules diffuse across a cell of diameter d in time τ. If the cell diameter is tripled, the diffusion time would

1. Increase to 9τ
2. Decrease to $\frac{\tau}{3}$
3. Increase to 3τ
4. Decrease to $\frac{\tau}{9}$.

(19.) Identify the figure which depicts a first order reaction.

1.



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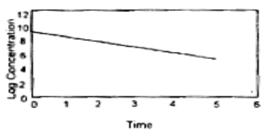

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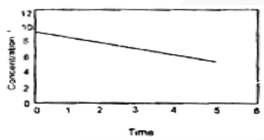

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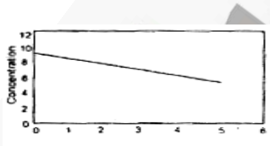

(20.) Which of the following particles has the largest range in a given medium if their initial energies are the same?

1. Alpha
2. Gamma
3. Positron
4. Electron

PART-B

(21.) Let $S = \left\{ A : A\left[a_{ij}\right]_{5\times 5}, a_{ij} = 0 \text{ or } 1\ \forall i, j,\ \sum_j a_{ij} = 1 \forall i \text{ and } \sum_j a_{ij} = 1 \forall j \right\}$.

Then the number of elements is S is

1. 5^2
2. 5^5
3. $5!$
4. 55

(22.) The number of 4 digit number with no two digits common is

1. 4536
2. 3024
3. 5040
4. 4823

(23.) Let D be a non zero $n \times n$ real matrix with $n \geq 2$. Which of the following implications is valid?

1. $\det(D) = 0$ implies ranks $(D) = 0$
2. $\det(D) = 1$ implies ranks $(D) \neq 1$
3. $\det(D) = 1$ implies ranks $(D) \neq 0$
4. $\det(D) = n$ implies ranks $(D) \neq 1$

(24.) Let $f_n(x) = x^{1/n}$ for $n \in [0, 1]$. Then

1. $\lim_{n\to\infty} f_n(x)$ exists for all $x \in [0, 1]$
2. $\lim_{n\to\infty} f_n(x)$ defines a continuous function on $[0, 1]$
3. $\{f_n\}$ converges uniformly on $[0, 1]$
4. $\lim_{n\to\infty} f_n(x) = 0$ for all $x \in [0, 1]$.

(25.) Let $A = \{x^2 : 0 < x < 1\}$ and $B = \{x^3 : 1 < x < 2\}$.
Which of the following statements is true?

1. There is a one to one, onto function from A to B
2. There is no one to one, onto function from A to B taking rationals to rationals.
3. There is no one to one function from A to B which is onto.
4. There is no onto function from A to B which is one to one.

(26.) Let ζ be a primitive fifth root of unity. Define

$$A = \begin{pmatrix} \zeta^{-2} & 0 & 0 & 0 & 0 \\ 0 & \zeta^{-1} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \zeta & 0 \\ 0 & 0 & 0 & 0 & \zeta^2 \end{pmatrix}$$

For a vector $v = (v_1, v_2, v_3, v_4, v_5) \in \mathbf{R}^5$, define $|v|_A = \sqrt{|vAv^T|}$ where v^T is transpose of v. If $w = (1, -1, 1, 1, -1)$, then $|w|_A$ equals

1. 0
2. 1
3. −1
4. 2

(27.) The number of elements in the set
$\{m : 1 \leq m \leq 1000, m$ and 1000 are relatively prime$\}$ is

1. 100
2. 250
3. 300
4. 400

(28.) The unit digit of 2^{100} is

1. 2
2. 4
3. 6
4. 8

(29.) The dimension of the vector space of all symmetric matrices of order $n \times n (n \geq 2)$ with real entries and trace equal to zero is

1. $\frac{(n^2 - n)}{2 - 1}$
2. $\frac{(n^2 - 2n)}{2 - 1}$
3. $\frac{(n^2 + n)}{2 - 1}$
4. $\frac{(n^2 + 2n)}{2 - 1}$

(30.) Let $I = \{1\} \cup \{2\} \subset \mathbf{R}$. For $x \in \mathbf{R}$, let $\varphi(x) = \text{dist}\,(x, I) = \inf\{|x - y| : y \in I\}$. Then

1. φ is discontinuous somewhere on $\mathbf{R}$
2. φ is continuous on $\mathbf{R}$ but not differentiable only $x = 1$
3. φ is continuous on $\mathbf{R}$ but not differentiable only $x = 1$ and 2
4. φ is continuous on $\mathbf{R}$ but not differentiable only $x = 1, \frac{3}{2}$ and 2.

(31.) The set $\left\{\frac{1}{n} \sin \frac{1}{n} : n \in \mathbf{N}\right\}$ has

1. One limit point and it is 0
2. One limit point and it is 1
3. One limit point and it is –1
4. Three limit points and these are –1, 0 and 1.

(32.) Using the fact that $\sum_1^\infty \frac{(-1)^{n+1}}{n} = \log 2, \sum_1^\infty \frac{(-1)^n}{n(n+1)}$ equals

1. $1 - 2\log 2$
2. $1 + 2\log 2$
3. $(\log 2)^2$
4. $-(\log 2)^2$

(33.) Let $f : \mathbf{C} \to \mathbf{C}$ be a complex valued function given by
$f(z) = u(x, y) + iv(x, y)$.
Suppose that $v(x, y) = 3xy^2$. Then

1. f cannot be holomorphic on $\mathbf{C}$ for any choice of u
2. f is holomorphic on $\mathbf{C}$ for a suitable choice of u
3. f is holomorphic on $\mathbf{C}$ for all choices of u
4. v is not differentiable as a function of x and y.

(34.) For $V = (V_1, V_2) \in \mathbf{R}^2$ and $W = (W_1, W_2) \in \mathbf{R}^2$,
Consider the determinant map
$\det : \mathbf{R}^2 \times \mathbf{R}^2 \to \mathbf{R}$ defined by
$\det(V, W) = V_1 W_2 - V_2 W_1$
Then the derivative of the determinant map at $(V, W) \in \mathbf{R}^2 \times \mathbf{R}^2$ evaluated on $(H, K) \in \mathbf{R}^2 \times \mathbf{R}^2$ is

1. $\det(H, W) + \det(V, K)$
2. $\det(H, K)$
3. $\det(H, V) + \det(W, K)$
4. $\det(V, W) + \det(K, W)$.

(35.) Let W be the vector space of all real polynomials of degree at most 3. Define $T : W \to W$ by $(Tp)(x) = p'(x)$ where p' is the derivative of p. The matrix of T in the basis $\{1, x, x^2, x^3\}$, considered as column vectors, is given by

1. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$
2. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{pmatrix}$
3. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
4. $\begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

(36.) The degree of the extension $\mathbf{Q}(\sqrt{2} + \sqrt{2})$ Over the field $\mathbf{Q}(\sqrt{2})$ is

1. 1
2. 2
3. 3
4. 6

(37.) The power series $\sum_{0}^{\infty} 2^{-n} z^{2n}$ converges if

1. $|z| \le 2$
2. $|z| < 2$
3. $|z| \ge \sqrt{2}$
4. $|z| < \sqrt{2}$

(38.) Consider a group G. Let Z(G) be its centre, i.e. $Z(G) = \{g \in G : gh = hg \text{ for all } h \in G\}$. For $n \in \mathbf{N}$, the set of positive integers, define

$J_n = \{(g_1, \ldots, g_n) \in Z(G) \times \ldots . \times Z(G) : g_1 \ldots . g_n = e\}$.

As a subset of the direct product group $Gx \ldots xG$ (n times direct product of the group G), J_n is

1. Not necessarily a subgroup
2. A subgroup but not necessarily a normal subgroup
3. A normal subgroup
4. Isomorphic to the direct product $Z(G) \times \ldots . \times Z(G)((n-1) \text{ times})$.

(39.) Let I_1 be the ideal generated by $x^4 + 3x^2 + 2$ and I_2 be the ideal generated by $x^3 + 1$ in $\mathbf{Q}[x]$. If $F_1 = \frac{\mathbf{Q}[x]}{I_1}$ and $F_2 = \frac{\mathbf{Q}[x]}{I_2}$, then

1. F_1 and F_2 are fields
2. F_1 is a field, but F_2 is not a field
3. F_1 is not a field while F_2 is a field
4. Neither F_1 nor F_2 is a field.

(40.) Let G be a group of order 77. Then the center of G is isomorphic to

1. $\mathbf{Z}_{(1)}$

2. $\mathbf{Z}_{(7)}$
3. $\mathbf{Z}_{(11)}$
4. $\mathbf{Z}_{(77)}$

(41.) Let P be a polynomial of degree N, with $N \geq 2$. Then the initial value problem $u'(t) = P(u(t)),\ u(0) = 1$ has always

1. A unique solution in $\mathbf{R}$
2. N number of distinct solution in $\mathbf{R}$
3. No solution in any interval containing 0 for some P.
4. A unique solution in an interval containing 0.

(42.) Consider the ODE

$u''(t) + P(t)u'(t) + Q(t)u(t) = R(t),\ t \in [0,\ 1]$

There exist continuous function P, Q and R defined on [0, 1] and two solutions u_1 and u_2 of the ODE such that the Wronskian W of u_1 and u_2 is

1. $W(t) = 2t - 1, \quad 0 \leq t \leq 1$
2. $W(t) = \sin 2\pi t, \quad 0 \leq t \leq 1$
3. $W(t) = \cos 2\pi t, \quad 0 \leq t \leq 1$
4. $W(t) = 1, \quad 0 \leq t \leq 1$

(43.) The number of characteristic curves of the PDE

$(x^2 + 2y)u_{xx} + (y^3 - y + u)u_{yy} + x^2(y-1)u_{xy} + 3u_x + u = 0$

passing through the point $x = 1, y = 1$ is

1. 0
2. 1
3. 2
4. 3

(44.) A general solution of the second order equation $4u_{xx} - u_{yy} = 0$ is of the form $u(x, y) =$

Where f and g are twice differentiable functions.

1. $f(x) + g(y)$
2. $f(x + 2y) + g(x - 2y)$
3. $f(x + 4y) + g(x - 4y)$
4. $f(4x + y) + g(4x - y)$

(45.) Consider the function $f(x) = e^{-x}$ and its Taylor approximation $g(x)$ of degree 3. For $x = \frac{1}{3}, g(x)$ is

1. Positive and less than 1
2. Negative and less than 2
3. Positive and greater than 1
4. Less than 1 but greater than 0.75

(46.) The variational problem of extremizing the functional

$$I(y(x)) = \int_0^{2\pi} \left[\left(\frac{d}{dx} y \right)^2 - y^2 \right] dx;\ y(0) = 1,\ y(2z) = 1$$

has

1. A unique solution
2. Exactly two solutions
3. An infinite number of solutions
4. No solution

(47.) For the Volterra type linear integral equation

$$\phi(x) = x + 2\int_0^x e^{x-\zeta}(\zeta)d\zeta,$$

the resolvent kernel $R(x,\zeta;2)$ of the kernel $e^{x-\zeta}$ is

1. $(e-\zeta)^2 e^{2(x-\zeta)}$
2. $(e-\zeta)e^{x-\zeta}$
3. $e^{3(x-\zeta)}$
4. $e^{(x-\zeta)}$

(48.) Which of the following is/are correct

1. A free particle in $\mathbf{R}^3$ can have infinite degrees of freedom
2. The number of degree of freedom of N particles is greater than 3N
3. A system of N particles with k constants has 3N + k degrees of freedom
4. A system consisting of three point masses connected by three rigid massless rod has six degrees of freedom.

(49.) A system of 5 identical units consists of two parts A and B which are connected in series. Part A has 2 units connected in parallel and part B has 3 units connected in parallel. All the 5 units function independently with probability of failure $\frac{1}{2}$. Then the reliability, of the system is

1. $\frac{31}{32}$
2. $\frac{11}{32}$
3. $\frac{1}{32}$
4. $\frac{21}{32}$

(50.) Suppose $X_1, X_2, ...$ is an i.i.d. sequence of random variables with common variance $\sigma^2 > 0$. Let $Y_n = \frac{1}{n}\sum_{i=1}^{n} X_{2i-1}$ and $Z_n = \frac{1}{n}\sum_{i=1}^{n} X_{2i}$.

Then the asymptotic distribution (as $n \to \infty$) of $\sqrt{n}(Y_n - Z_n)$ is

1. $N(0,\ 1)$
2. $N(0,\ \sigma^2)$
3. $N(0,\ 2\sigma^2)$
4. Degenerate at 0

(51.) Consider an aperiodic Markov chain with state space S and with stationary transition probability matrix $P = ((p_{ij}), ij \in S$. Let the n-step transition probability matrix be denoted by $P^n = ((p_{ij}^n), i, j \in S$. Then which of the following statements is true?

1. $\lim_{n\to\infty} p_{ii}^n = 0$ Only if i is transient
2. $\lim_{n\to\infty} p_{ii}^n > 0$ and only if i is recurrent
3. $\lim_{n\to\infty} p_{ij}^n = \lim_{n\to\infty} p_{jj}^n$ if i and j are in the same communicating class.
4. $\lim_{n\to\infty} p_{ij}^n = \lim_{n\to\infty} p_{ii}^n$ if i and j are in the same communicating class.

(52.) Suppose X is a random variable with $E(X) = Var(X)$. Then the distribution of X

1. Is necessarily Poisson
2. Is necessarily Exponential
3. Is necessarily Normal
4. Cannot be identified from the given data.

(53.) Let $x = 10$ be an observation on the hypergeometric random variable X, namely

$$P(X = x) = \frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}}, \; x = 0, 1, \ldots, \quad \min\{m, n\} \text{ and } n - x \le N - m$$

Where $m = 40, n = 30$ and N is an unknown parameter. The maximum likelihood estimator of N is

1. 75
2. 120
3. 60
4. Not unique.

(54.) Let $X_1, X_2, \ldots, X_n,\ n \ge 2,$ be i.i.d. observations from $N(0,\ \sigma^2)$ distribution, where $0 < \sigma^2 < \infty$ is an unknown parameter. Then the uniformly minimum variance unbiased estimate for σ^2 is

1. $\frac{1}{n}\sum_{i=1}^{n} X_i^2$
2. $\frac{1}{n-1}\sum_{i=1}^{n} X_i^2$
3. $\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$
4. $\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$

(55.) Suppose that we have i.i.d. observations $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n), n \ge 3,$ where X_i and Y_i are independent normal random variables. Consider $\tau =$ the sample Kendall's rank correlation coefficient computed from this data. Then which of the following is correct?

1. $P(\tau > 0) > \frac{1}{2}$
2. $P(\tau < 0) > \frac{1}{2}$
3. $E(\tau) = 0$
4. $E(\tau) \ne 0$

(56.) The reaction time to a stimulus X (in seconds) is distributed normally in
group 1 with mean 2 and variance 8;
group 2 with mean 4 and variance 1.
The two groups appear in equal properties.
x is an observable value of X. the best discriminant function (in the sense of minimizing misclassification probabilities) is to classify into group

1. 2 if $x > 3$; otherwise in group 1
2. 1 if $x > 3$; otherwise in group 2
3. 2 if $0 \le x \le \frac{8}{3}$; otherwise in group 1
4. 1 if $0 \le x \le \frac{8}{3}$; otherwise in group 2.

(57.) Batteries for torch lights are packed in boxes of 10 and a lot contains 10 boxes. A quality inspector randomly chooses a box and then checks two batteries selected randomly without replacement from that box. The lot will be rejected if any one of the two chosen batteries turns out to be defective. Suppose that 9 of the 10 boxes in the lot contain no defective batteries and only one box contains 2 defective ones. What is the probability that the lot will NOT be passed by the Inspector?

1. $\frac{197}{4950}$

2. $\frac{98}{2475}$
3. $\frac{8}{225}$
4. $\frac{17}{450}$

(58.) To examine whether two different skin creams, A and B, have different effect on the human body n randomly chosen persons were enrolled in a clinical trial. Then cream A was applied to one of the randomly chosen arms of each person, cream B to the other. What kind of a design is this?
1. Completely Randomized Design
2. Balanced Incomplete Block Design
3. Randomized Block Design
4. Latin Square Design

(59.) Consider the LP problem
Maximum $x_1 + x_2$
Subject to
$$x_1 - 2x_2 \le 10$$
$$x_2 - 2x_1 \le 10$$
$$x_1, x_2 \ge 0$$
Then
1. The LP problem admits an optimal solution
2. The LP problem is unbounded
3. The LP problem admits no feasible solution
4. The LP problem admits a unique feasible solution.

(60.) Let $X(t)$ be the number of customers in an M/M/1 queueing system with arrival rate 3 and service rate 6. Which of the following is true?
1. $\lim_{t\to\infty} P(X(t) \ge 5) = 0$
2. $\lim_{t\to\infty} P(X(t) \ge 5) = \frac{1}{32}$
3. $\lim_{t\to\infty} P(X(t) \ge 5) = \frac{31}{32}$
4. $\lim_{t\to\infty} P(X(t) \ge 5) = 1$

PART-C

Unit-I

(61.) Consider the function
$f(x) = |\cos x| + |\sin(2-x)|$
At which of the following points is f not differentiable?
1. $\left\{(2n+1)\frac{\pi}{2} : n \in \mathbf{Z}\right\}$
2. $\{n\pi : n \in \mathbf{Z}\}$
3. $\{n\pi + 2 : n \in \mathbf{Z}\}$
4. $\left\{\frac{n\pi}{2} : n \in \mathbf{Z}\right\}$

(62.) Which of the following subsets of $\mathbf{R}^2$ are convex?
1. $\{(x, y) : |x| \le 5\ |y| \le 10\}$

2. $\{(x, y) : x^2 + y^2 = 1\}$
3. $\{(x, y) : y \geq x^2\}$
4. $\{(x, y) : y \leq x^2\}$

(63.) Which of the following is/are metrics on $\mathbf{R}$?
1. $d(x, y) = \min(x, y)$
2. $d(x, y) = |x - y|$
3. $d(x, y) = |x^2 - y^2|$
4. $d(x, y) = |x^3 - y^3|$

(64.) Let X denote the two-point set {0, 1} and write $X_j = \{0, 1\}$ for every $j = 1, 2, 3, ...$ Let $Y = \prod_{j=1}^{\infty} X_j$.

Which of the following is/are true?
1. Y is a countable set
2. Card Y = card, [0, 1]
3. $\bigcup_{n=1}^{\infty}\left(\prod_{j=1}^{\infty} X_j\right)$ is uncountable
4. Y is uncountable.

(65.) Which of the following is/are correct?
1. $n \log\left(1 + \frac{1}{n+1}\right) \to 1$ as $n \to \infty$
2. $(n+1) \log\left(1 + \frac{1}{n+1}\right) \to 1$ as $n \to \infty$
3. $n^2 \log\left(1 + \frac{1}{n}\right) \to 1$ as $n \to \infty$
4. $n \log\left(1 + \frac{1}{n^2}\right) \to 1$ as $n \to \infty$

(66.) If $\{x_n\}$ and $\{y_n\}$ are sequences of real numbers, which of the following is/are true?
1. $\limsup_n (x_n + y_n) \leq \limsup_n x_n + \limsup_n y_n$
2. $\limsup_n (x_n + y_n) \geq \limsup_n x_n + \limsup_n y_n$
3. $\liminf_n (x_n + y_n) \leq \liminf_n x_n + \liminf_n y_n$
4. $\liminf_n (x_n + y_n) \geq \liminf_n x_n + \liminf_n y_n$

(67.) Let $\{f_n\}$ be a sequence of integrable functions defined on an interval $[a, b]$. Then
1. If $f_n(x) \to 0$ a.e., then $\int_a^b f_n(x)dx \to 0$
2. If $\int_a^b f_n(x)dx \to 0$, then $f_n(x) \to 0$ a.e.
3. If $f_n(x) \to 0$ a.e. and each f_n is a bounded function, then $\int_a^b f_n(x)dx \to 0$
4. If $f_n(x) \to 0$ a.e. and the f_n's are uniformly bounded, then $\int_a^b f_n(x)dx \to 0$

(68.) For $x = (x_1, x_2, ..., x_4) \in \mathbf{R}^d$, and $p \geq 1$, define

$\|x\|_p = \left(\sum_{j=1} |x_j|^p \right)^{1/p}$ and

$\|x\|_\infty = \max \{ |x_j| : j = 1, 2, ...d \}$

Which of the following inequalities holds for all $x \varepsilon \mathbf{R}^d$?

1. $\|x\|_1 \ge \|x\|_2 \ge \|x\|_\infty$
2. $\|x\|_1 \ge d \|x\|_\infty$
3. $\|x\|_1 \le \sqrt{d} \|x\|_\infty$
4. $\|x\|_1 \le \sqrt{d} \|x\|_2$

(69.) Consider the map $f : \mathbf{R}^2 \to \mathbf{R}^2$ defined by $f(x, y) = (3x - 2y + x^2, 4x + 5y + y^2)$. Then

1. f is discontinuous at (0, 0)
2. f is continuous at (0, 0) and all directional derivatives exists at (0, 0)
3. f is differentiable at (0, 0) but the derivative $Df(0, 0)$ is not invertible
4. f is differentiable at (0, 0) and the derivative $Df(0, 0)$ is invertible.

(70.) Which of the following sets are dense in $\mathbf{R}$ with respect to the usual topology.

1. $\{(x, y) \in \mathbf{R}^2 : x \in \mathbf{N}\}$
2. $\{(x, y) \in \mathbf{R}^2 : x + y \text{ is a rational number}\}$
3. $\{(x, y) \in \mathbf{R}^2 : x^2 + y^2 = 5\}$
4. $\{(x, y) \in \mathbf{R}^2 : xy \ne 0\}$

(71.) Let $F = \{ f : \mathbf{R} \to \mathbf{R} : |f(x) - f(y)| \le K |(x - y)|^\alpha \}$

For all $x, y \in \mathbf{R}$ and for some $\alpha > 0$ and some $K > 0$.

Which of the following is/are true?

1. Every $f \in F$ is continuous
2. Every $f \in F$ is uniformly continuous
3. Every differentiable function f is in F
4. Every $f \in F$ is differentiable

(72.) Let $a_{ij} = a_i a_j$, $1 \le i, j \le b$, where $a_1, ..., a_n$ are real numbers. Let $A = ((a_{ij}))$ be the $n \times n$ matrix $((a_{ij}))$. Then

1. It is possible to choose $a_1, ..., a_n$ so as to make the matrix A non singular
2. The matrix A is positive definite if $(a_1, ..., a_n)$ is a non zero vector
3. The matrix A is positive semi definite for all $(a_1, ..., a_n)$
4. For all $(a_1, ..., a_n)$, zero is an eigenvalue of A.

(73.) Suppose A, B are $n \times n$ positive definite matrices and I be the $n \times n$ identity matrix. Then which of the following are positive definite.

1. $A + B$
2. ABA^*
3. $A^2 + I$
4. AB

(74.) Let T be a linear transformation on the real vector space $\mathbf{R}^n$ over $\mathbf{R}$ such that $T^2 = \lambda T$ for same $\lambda \in \mathbf{R}$. Then

1. $\|Tx\| = |\lambda| \|x\|$ for all $x \in \mathbf{R}^n$
2. If $\|Tx\| = \|x\|$ for some non zero vector $x \in \mathbf{R}^n$, then $\lambda = \pm 1$
3. $T = \lambda I$ where I is the identity transformation on $\mathbf{R}^n$

4. If $\|Tx\| > \|x\|$ for a non zero vector $x \in \mathbf{R}^n$, then T is necessarily singular.

(75.) Let M be the vector space of all 3 × 3 real matrices and let

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Which of the following are subspaces of M?

1. $\{X \in M : XA = AX\}$
2. $\{X \in M : X + A = A + X\}$
3. $\{X \in M : \text{trace}\,(AX) = 0\}$
4. $\{X \in M : \det\,(AX) = 0\}$

(76.) Let $W = \{p(B) : p$ is a polynomial with real coefficients}, where $B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

The dimension d of the vector space W satisfies

1. $4 \le d \le 6$
2. $6 \le d \le 9$
3. $3 \le d \le 8$
4. $3 \le d \le 4$

(77.) Let N be a 3 × 3 non zero matrix with the property $N^3 = 0$. Which of the following is/are true?

1. N is not similar to a diagonal matrix
2. N is similar to a diagonal matrix
3. N has one non-zero eigenvector
4. N has three linearly independent eigenvector.

(78.) Let $x, y \in \mathbf{C}^n$. Consider $f(x, y) = \sup\limits_{\theta, \varphi} \left\| e^{i\theta} x - e^{i\varphi} y \right\|_2$, $\theta, \phi \in \mathbf{R}$.

Which of the following is/are correct?

1. $f(x, y) \le \|x\|^2 + \|y\|^2 - 2\,\text{Re}\,|\langle x, y\rangle|$
2. $f(x, y) \le \|x\|^2 + \|y\|^2 + 2\,\text{Re}\,|\langle x, y\rangle|$
3. $f(x, y) = \|x\|^2 + \|y\|^2 + 2|\langle x, y\rangle|$
4. $f(x, y) \ge \|x\|^2 + \|y\|^2 - 2\,\text{Re}\,|\langle x, y\rangle|$

Unit-II

(79.) Let $\mathbf{D} = \{z \in \mathbf{C} : |z| < 1\}$ be the unit disc. Let $f : \mathbf{D} \to \mathbf{C}$ be an analytic function satisfying $f\left(\frac{1}{n}\right) = \frac{2n}{3n+1}$ for $n \ge 1$. Then

1. $f(0) = \frac{2}{3}$
2. f has a simple pole at $z = -3$
3. $f(3) = \frac{1}{3}$
4. No such f exists.

(80.) Let f be an entire function. If $\text{Re}\,f$ is bounded then

1. $Im\ f$ is constant
2. f is constant
3. $f \equiv 0$

4. f' is a non zero constant

(81.) Let $f:\mathbf{D}\to\mathbf{D}$ be holomorphic with $f(x)=\frac{1}{2}$ and $f\left(\frac{1}{2}\right)=0$, where $\mathbf{D}=\{z:|z|\le 1\}$.

Which of the following is correct?

1. $|f'(0)|\le\frac{3}{4}$
2. $\left|f'\left(\frac{1}{2}\right)\right|\le\frac{4}{3}$
3. $|f'(0)|\le\frac{3}{4}$ and $\left|f'\left(\frac{1}{2}\right)\right|\le\frac{4}{3}$
4. $f(z)=z, z\in\mathbf{D}$

(82.) Define

$H^+=\{z\in\mathbf{C}:y>0\}$

$H^-=\{z\in\mathbf{C}:y<0\}$

$L^+=\{z\in\mathbf{C}:x>0\}$

$L^-=\{z\in\mathbf{C}:x<0\}$

The function $f(z)=\frac{z}{3z+1}$

1. Maps H^+ onto H^+ and H^- onto H^-
2. Maps H^+ onto H^- and H^- onto H^+
3. Maps H^+ onto L^+ and H^- onto L^-
4. Maps H^+ onto L^- and H^- onto L^+

(83.) At $z=0$ the function $f(z)=\frac{e^z+1}{e^z-1}$ has

1. A removable singularity
2. A pole
3. An essential singularity
4. The residue of $f(z)$ at $z=0$ is 2.

(84.) Let $H=\{e,(1,2)(3,4)\}$ and $K=\{e,(1,2)(3,4),(1,3)(2,4),(1,4)(2,3)\}$ be subgroups of S_4, where e denotes the identify element of S_4. Then

1. H and K are normal subgroups of S_4
2. H is normal in K and K is normal in A_4
3. H is normal in A_4 but not normal in S_4
4. K is normal in S_4, but H is not.

(85.) Let $\langle p(x)\rangle$ denote the ideal generated by the polynomial $p(x)$ in $\mathbf{Q}[x]$. If $f(x)=x^3+x^2+x+1$ and $g(x)=x^3-x^2+x-1$, then

1. $\langle f(x)\rangle+\langle g(x)\rangle=\langle x^3+x\rangle$
2. $\langle f(x)\rangle+\langle g(x)\rangle=\langle f(x)\cdot g(x)\rangle$
3. $\langle f(x)\rangle+\langle g(x)\rangle=\langle x^2+1\rangle$
4. $\langle f(x)\rangle+\langle g(x)\rangle=\langle x^2-1\rangle$

(86.) Let I_1 be the ideal generated by x^2+1 and I_2 be the ideal generated by x^3-x^2+x-1 in $\mathbf{Q}[x]$.

If $R_1=\frac{\mathbf{Q}[x]}{I_1}$ and $R_2=\frac{\mathbf{Q}[x]}{I_2}$, then

1. R_1 and R_2 are fields

2. R_1 is a field and R_2 is not a field
3. R_1 is an integral domain, but R_2 is not an integral domain
4. R_1 and R_2 are not integral domains.

(87.) Let $G = \mathbf{Z}_{10} \times \mathbf{Z}_{15}$. Then
1. G contains exactly one element of order 2
2. G contains exactly 5 elements of order 3
3. G contains exactly 24 elements of order 5
4. G contains exactly 24 elements of order 10

(88.) The space $C[0, 1]$ of continuous functions on $[0, 1]$ is complete with respect to which of the following
1. $\|f\|_\infty = \sup\{|f(x)| : x \in [0, 1]\}$
2. $\|f\|_2 = \left(\int_0^1 |f(x)|^2 \, dx\right)$
3. $\|f\|_{\infty, 1/2} = \|f\|_\infty + \left|f\left(\frac{1}{2}\right)\right|$
4. $\|f\|_\infty$ and $\|f\|_{\infty, 1/2}$

(89.) Consider the set
$X = (-\infty, 0] \cup \left\{\frac{1}{n} : n \in \mathbf{N}\right\} \subseteq \mathbf{R}$ with the subspace topology. Then
1. 0 is an isolated point
2. (–2, 0] is an open set
3. 0 is a limit point of the subset $\left\{\frac{1}{n} : n \in \mathbf{N}\right\}$
4. (–2, 0)is an open set.

(90.) Consider three subsets of $\mathbf{R}^2$, namely
$A_1 = \{(x, y) : x^2 + y^2 \le 1\}$
$A_2 = \{(1, y) : y \in \mathbf{R}\}$
$A_3 = \{(0, 2)\}$
Then there always exists a continuous real-valued function f on $\mathbf{R}^2$ such that $f(x) = a_j$ for $x \in A_j$, $j = 1, 2, 3$
1. If and only if at least two of the numbers a_1, a_2, a_3 are equal
2. If $a_1 = a_2 = a_3$
3. For all real values of a_1, a_2, a_3
4. If and only if $a_1 = a_2$

Unit-III

(91.) The Green's function $G(x, \zeta)$, $0 \le x, \zeta \le 1$ of the boundary value problem
$y'' + \lambda y = 0,$
$y(0) = 0 = y(1)$
is
1. Symmetric in x and ζ
2. Continuous at $x = \zeta$
3. $\left.\frac{\partial G(x, \zeta)}{\partial x}\right|_{x=\zeta^-} - \left.\frac{\partial G(x, \zeta)}{\partial x}\right|_{x=\zeta^+} = -1$

4. $\left.\frac{\partial G(x,\zeta)}{\partial x}\right|_{x=\zeta^-} - \left.\frac{\partial G(x,\zeta)}{\partial x}\right|_{x=\zeta^+} = 1$

(92.) For the boundary value problem,
$y'' + \lambda y = 0,$
$y(-\pi) = y(\pi),$
$y'(-\pi) = y'(\pi),$
to each eigen value $\lambda,$ there corresponds
1. Only one eigen function
2. Two eigen functions
3. Two linearly independent eigen functions
4. Two orthogonal eigen functions

(93.) Let $y_1(x)$ and $y_2(x)$ form a fundamental set of solutions to the differential equation
$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)\,y = 0, \quad 0 \le x \le b,$
Where $p(x)$ and $q(x)$ are continuous in $[a, \ b]$, and x_0 is a point in $(a, \ b)$. Then
1. Both $y_1(x)$ and $y_2(x)$ cannot have a local maximum at x_0.
2. Both $y_1(x)$ and $y_2(x)$ cannot have a local minimum at x_0.
3. $y_1(x)$ cannot have a local maximum at x_0 and $y_2(x)$ cannot have local minimum at x_0 simultaneously.
4. Both $y_1(x)$ and $y_2(x)$ cannot vanish at x_0 simultaneously.

(94.) A general solution of the PDE
$uu_x + yu_y = x$ is of the form
1. $f\left(u^2 - x^2, \frac{y}{x+u}\right) = 0,$ where $f : \mathbf{R}^2 \to \mathbf{R}$ is C^1 and $\nabla f \ne (0, \ 0)$ at every point
2. $u^2 = g\left(\frac{y}{x+u}\right) + x^2, \ g \in C^1(\mathbf{R})$
3. $f\left(u^2 + x^2\right) = 0, f \in C^1(\mathbf{R})$
4. $f(x+y) = 0, \ f \in C^1(\mathbf{R})$

(95.) The PDE
$$\left.\begin{aligned} &u_{xx} + u_{yy} + \lambda u = 0, && 0 < x, y < 1 \\ &u(x,0) = u(x,1) = 0, && 0 \le x \le 1 \\ &u(0,y) = u(1,y) = 0, && 0 \le y \le 1 \end{aligned}\right\}$$
has
1. A unique solution u for any $\lambda \in \mathbf{R}$
2. Infinitely many solutions for some $\lambda \in \mathbf{R}$
3. A solution for countably many values of λ
4. Infinitely many solutions for all $\lambda \in \mathbf{R}$.

(96.) The Cauchy problem
$$\left.\begin{aligned} u_x(x,y) + u_y(x,y) &= 0 && \text{for } (x,y) \in \mathbf{R}^2 \\ u(x,x) &= && \text{for } all \ x \in \mathbf{R} \end{aligned}\right\}$$
has
1. A unique solution
2. A family of straight lines as characteristics
3. Solution which vanishes at (2, 1)
4. Infinitely many solutions.

(97.) Consider a linear system $Ax = b$ with a computed solution x_C; the error and the residue are defined, respectively by

$e = x - x_c$

$r = Ax - Ax_c$

Then

1. A small error necessarily implies a small residue
2. The error can be larger with relatively small residue
3. The error can be small with relatively large residue
4. The error and the residue are always equal.

(98.) Consider the iteration function for Newton's method

$$g(x) = x - \frac{f(x)}{f'(x)}$$

and its application to find (approximate) square root of 2, starting with $x_0 = 2$. Consider the first and the second iterates x_1 and x_2, respectively; then

1. $1.5 < x_1 \le 2$
2. $1.5 \le x_1 < 2$
3. $x_1 \le 1.5;\ x_2 \le 1.5$
4. $x_1 = 1.5;\ x_2 < 1$

(99.) In the Ritz method, seeking an extremum of the functional

$$I(y) = \int_{x_0}^{x_1} F\left(x, y, \frac{dy}{dx}\right)dx;\ \ y(x_0) = a, y(x_1) = b,$$

The coordinate function/or the admissible function $\phi_i(x), i = 1, 2, ...$ defined on $[x_0,\ x_1]$ must be

1. Linearly independent
2. Continuous
3. Smooth
4. Linearly independent, smooth and the functional be considered not along admissible curves $y = y(x)$ but only along all possible linear combinations of admissible functions.

(100.) The integral equation, involving a parameter λ,

$\phi(x) = \cos xz + \lambda\int_0^{\pi} \cos(x+\zeta)\,d\zeta$ has

1. A unique solution if $\lambda = 1$, and an infinite number of solution if $\lambda = \frac{2}{\pi}$
2. A unique solution if $\lambda = -1$, and an infinite number of solution if $\lambda = -\frac{2}{\pi}$
3. A unique solution if $\lambda \ne \frac{2}{\pi}$
4. No solution if $\lambda = \pm\frac{2}{\pi}$

(101.) Consider the force free motion of a rigid body about a fixed point 0. Suppose 3A, 5A and 6A are the principal moments of inertia at 0, and initially the angular velocity has components $\omega_1 = \sqrt{5}, \omega_2 = 0, \omega_3 = \sqrt{5}$ about the corresponding principal axes; if the body ultimately rotates about the mean axis, then

1. $\omega_1^2 + \omega_2^2 = 5$
2. $5\omega_2^2 + g\omega_1^2 = 45$
3. $\omega_3^2 = \omega_1^2$
4. $\omega_2^2 \ne \omega_1^2$

(102.) Using Euler's dynamical equation for force free motion of a rigid body, symmetrical about the Z-principal axis, with angular velocity $\bar{\omega} = (\omega_1, \omega_2, \omega_3)$, where $\omega_i,\ i = 1, 2, 3,$ are the components along the three principal axes, it follows that

1. $\omega_i =$ constant
2. $\omega_2 = a \sin(\lambda t + b)$ with a $a, \lambda,$ and b as constant
3. $\omega_3 =$ constant
4. $\omega_1^2 + \omega_2^2$ constant

Unit-IV

(103.) Which of the following is/are cumulative distribution function(s) (c.d.f.) of random variable(s)?

1. $F_1(x) = \begin{cases} 0, & x \le 0 \\ e^{-x}, & x > 0 \end{cases}$
2. $F_2(x) = \begin{cases} 0, & x \le 0 \\ 1 - e^{-x}, & x > 0 \end{cases}$
3. $F_3(x) = \begin{cases} 0, & x \le 0 \\ 1, & x > 0 \end{cases}$
4. $F_4(x) = \begin{cases} 0, & x < 0 \\ 1/2, & 0 \le x < 1 \\ 1, & x \ge 0 \end{cases}$

(104.) Let X be a random variable taking values in a set E. Let $P(x > a + b \mid X > a) = P(X > b)$ for all $a, b, \in E.$

Then which of the following is a possible distribution of X?

1. Poisson
2. Geometric
3. Long-normal
4. Exponential

(105.) Let $\{X_n\}$ be a stationary Markov chain such that

$P(X_{i+1} = 1 \mid X_i = 1) = p_1 = 1 - P(X_{i+1} = 0 \mid X_i = 1)$

$P(X_{i+1} = 1 \mid X_i = 0) = p_0 = 1 - P(X_{i+1} = 0 \mid X_i = 0)$,

and $P(X_1 = 1) = \pi_1 = 1 - P(X_1 = 0)$

Then

1. $\pi_1 = p_1$
2. $\pi_1 = p_0$
3. $\pi_1 = \dfrac{p_0}{1 - p_1 + p_0}$
4. $\pi_1 = \dfrac{1}{2}$

(106.) Suppose X and Y are independent N(0, 1) random variables.

Let $U = \dfrac{X}{Y}$ and $V = \dfrac{X}{|Y|}$. Then

1. U and V are independent
2. U and V have the same distribution
3. $P(U = V) = \dfrac{1}{2}$
4. $P(U < V) = \dfrac{1}{2}$

(107.) Suppose $X_1, X_2, ...$ is a sequence of i.i.d. random variables where
$P(X_i = 1) = p = 1 - P(X_i = 0),\ i = 1, 2, ...$

Let $Z = \frac{1}{500}\sum_{i=1}^{500} X_i$ and $\alpha = P(|Z - p| > 0.1)$

Then for all p

1. $\alpha \le .1$
2. $\alpha \le .05$
3. $\alpha > .01$
4. $\alpha = 0$

(108.) Suppose $X_1 \sim U(0,\ \theta),\ X_2 \sim U(0, 1+\theta)$ and X_1 and X_2 are independent. Then

1. $\min\{X_1, X_2\}$ is sufficient for θ
2. $\max\{X_1, X_2\}$ is sufficient for θ
3. $\max\{X_1, X_2 - 1\}$ is sufficient for θ
4. $\max\{X_1 + 1, X_2\}$ is sufficient for θ

(109.) Suppose that we have $n \ge 1$ i.i.d. observations $X_1, X_2, ..., X_n$ each with a common $N(\mu,\ 1)$ distribution where $\mu \ge 0$ is unknown parameter. Then

1. The maximum likelihood estimate and the uniformly minimum variance unbiased estimate for μ are the same
2. The minimum variance unbiased estimate for μ is a consistent estimate.
3. For any unbiased estimate for μ, there is another estimate for μ with a smaller mean squared error.
4. The maximum likelihood estimate for μ has smaller mean squared error than the estimate obtained by the method of moments.

(110.) Let $X_1, X_2, ...$ be i.i.d. observations from $N(\mu, \sigma^2)$ distribution with $-\infty < \mu < +\infty$ and $0 < \sigma^2 < \infty$ as unknown parameters.
Then

1. Sample mean is an unbiased estimate for μ but sample median is not an unbiased estimate for μ.
2. Both sample mean and sample median are unbiased estimate for μ.
3. Sample mean has smaller variance than sample median.
4. Sample mean has smaller mean squared error than sample median.

(111.) Suppose $X \sim N(0,\ \sigma^2), Y$ has the exponential distribution with mean $2\sigma^2$ and, X and Y are independent. We want to test at level α
$H_0 : \sigma^2 \le 1$ versus $H_1 : \sigma^2 > 1$
Then

1. UMP test does not exist
2. UMP test rejects H_0 when $X^2 + Y$ is large
3. UMP test is a chi-square test
4. UMP test is a t-test.

(112.) Suppose that the probability distribution of a discrete random variable X under two possible parameter values is as follows:

Parameter	1	2	3	4
θ_1	.01	.04	.05	.90
θ_2	.80	.10	.05	.05

Test $H_0 : \theta = \theta_1$ versus $H_1 : \theta = \theta_2$ at level $\alpha = 0.05$.
Then the most powerful test

1. Reject H_0 if $x = 1$ or $x = 2$

2. Reject H_0 if $x = 3$
3. Has power larger than 0.85
4. Has power 0.5

(113.) In a Bayesian estimation problem of the Poisson mean $\lambda,$ a gamma prior (with density proportional to $e^{-\beta\lambda}\lambda^{\alpha-1}$) is formulated. There is a sample of size n from the Poisson and the sample mean is $\bar{x}$. The posterior distribution of λ is

1. A gamma distribution
2. A Poisson distribution
3. Has mean $= \dfrac{n\bar{x}+\alpha}{n+\beta}$
4. Has mean $= (n\bar{x}+\alpha)(n+\beta)$

(114.) Random variables X_1, X_2, X_3 are such that correlation (X_1, X_2) = correlation (X_2, X_3) = correlation $(X_3, X_1) = \rho$

1. ρ cannot be negative
2. ρ can take any value between –1 and +1
3. ρ $\rho \geq -0.5$
4. ρ is either +1 or –1

(115.) Consider a linear model with four observations X_1, X_2, X_3, X_4 such that

$E(X_1) = A + B + C;$
$E(X_2) = A;$
$E(X_3) = B;$
$E(X_4) = A - B - C$

[Where A, B, C, D are parameters]. Then

1. B + C is not estimable
2. A, B, C are all estimable
3. A + B + C is estimable
4. X_2 is the Best Linear unbiased estimate of A.

(116.) In a survey of a population of $N = nk$ units, a sample of n units is to be drawn by systematic sampling with a random start between 1 and k and selecting every k^{th} units. Then

1. The sample mean is an unbiased estimate of the population mean
2. The variance of the sample mean cannot be estimated under this design
3. If the N population units have been arranged at random, then the sample is equivalent to a simple random sample with replacement
4. If the N population units have been arranged at random, then the sample is equivalent to a simple random sample without replacement.

(117.) Let **D** be a balanced incomplete block design with usual parameters v, b, r, k, λ. Which of the following statements is true?

1. **D** is connected if $k \geq 2$.
2. The variance of the best linear unbiased estimator of an elementary treatment contrast under **D** is proportional to $\dfrac{2}{r}$
3. The convariance between the best linear unbiased estimators of a pair of orthogonal treatment contrasts under **D** is zero.
4. The efficiency factor of **D** relative to a randomized (complete) block design with replication r is strictly smaller than unity.

(118.) Suppose that we have a data set consisting of 25 observations, where each value is either 5 or 10.

1. The mean of the data cannot be larger than the median.
2. The mean of the data cannot be smaller than the median.

3. The mean and the median for the data will be the same only if the variance of the data is zero
4. The mean and the median for the data will be different only if the range is 5.

(119.) Suppose that the LP problem

Maximize $c^T x$

Subject to

$Ax \le b$

$x \ge 0$

Admits a feasible solution and the dual minimize $b^T y$

Minimizes $b^T y$

Subject to $A^T y \ge c$

$y \ge 0$

Admits a feasible solution y_0.

Then

1. The dual admits an optimal solution
2. Any feasible solution x_0 of the primal and y_0 of the dual satisfies $b^T y_0 \le c^T x_0$
3. The dual problem is unbounded
4. The primal problem admits an optimal solution.

(120.) Let $X(t)$ be the number of customers in an M/M/1 queuing system with arrival rate $\lambda > 0$ and service rate $\mu > 0$.

It is known that $\lim_{t\to\infty} P(X(t)=1) = \frac{1}{4}$.

Which of the following is true?

1. $\lim_{t\to\infty} E(X(t)=1) = \frac{1}{3}$
2. $\lim_{t\to\infty} E(X(t)=1) = \frac{\lambda}{\mu}$
3. $\lim_{t\to\infty} Var(X(t)=1) = \frac{1}{9}$
4. $\lim_{t\to\infty} Var(X(t)=1) = \left(\frac{\lambda}{\mu}\right)^2$